Final Exam - December 14, 2006
Math 114

Name:

Instructor:

Teaching Assistant:

Recitation Day/Time:

There are nine questions on this examination. Some have multiple parts. **It is important to show your work and justify each statement**. You will receive partial credit for substantial progress towards the answers. You will lose partial credit for answers that are not justified. **Please write legibly.** No calculators, books, or notes may be used except for one two-sided 8.5"x11" sheet of notes.

The time limit for this exam is 120 minutes.

Good luck!

Question	Points
1	/10
2	/10
3	/10
4	/10
5	/10
6	/10
7	/10
8	/10
9	/10
TOTAL	/90

1. Solve the initial value problem:

$$\frac{dy}{dx} = -\sqrt{y}\sin x, \quad y(0) = 4.$$

2. Solve the following differential equation:

$$2y' + \frac{4}{x}y = \left(\frac{\sqrt{2}\cos x}{x}\right)^2.$$

3. (a) A particle is traveling upwards along the y-axis. It starts at the origin at time $t = 0$ with speed equal to 2 units per second. At each time t, the magnitude of its acceleration is equal to its distance from the origin, and the direction of the acceleration is along the positive y-axis. Let $y(t)$ be the distance of the particle from the origin at time t.

   i. Write down an initial-value problem that is satisfied by $y(t)$.

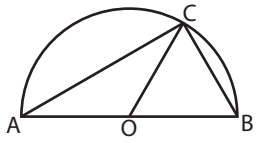
   ii. Solve the initial-value problem to find $y(t)$.

(b) Find the most general solution, $y(t)$, to the differential equation:

$$y'' - y = te^t.$$

4. (a) Consider the quadrilateral with vertices $A = (1, 0)$, $B = (5, 3)$, $C = (8, 7)$, and $D = (4, 4)$. Show that the line segment $\overline{AC}$ is perpendicular to the line segment $\overline{BD}$.

(b) Use vectors to show that any angle inscribed in a semicircle is a right angle. That is, show that $\angle ACB$ is a right angle where C is any point on the circle, AB is a diameter, and O is the center.



5. (a) Find the length of the curve parameterized by

$$\mathbf{r}(t) = (\cos^3 t)\mathbf{i} + (\sin^3 t)\mathbf{j}, \qquad 0 \leq t \leq \frac{\pi}{2}.$$

(b) Find the length of the curve that is the graph of

$$y = \frac{2}{3}\left(x^2 - 1\right)^{3/2}, \qquad 1 \leq x \leq 3.$$

6. Two particles, A and B, move through space. At time t (measured in seconds), the position of A is

$$\langle 2+t, -2+2t, 3-t \rangle$$

and the position of B is

$$\langle 2-4t+t^2, -2+7t-t^2, 3-6t+t^2 \rangle.$$

(Each component is measured in feet.)

(a) Is there a time $t > 0$ at which the particles are moving in the same direction?

(b) At a certain time $t > 0$ the two particles collide. Find this time.

(c) What is the magnitude of the acceleration of B at the time of collision?

(d) How far does A travel in the one second before the collision?

7. Material for the top of a rectangular box costs \$3 per square foot. Material for the bottom and four sides costs \$5 per square foot. What is the largest volume the box can have if the total cost of material for the box is \$96?

8. Evaluate

$$\iiint_E e^{\sqrt{x^2+y^2+z^2}}\, dV$$

where E is the solid region in the first octant $\{(x, y, z)|x \geq 0, y \geq 0, z \geq 0\}$ enclosed by $x^2 + y^2 + z^2 = 9$.

9. Consider $g : \mathbb{R}^2 \to \mathbb{R}$ defined by:

$$g(x,y) = \begin{cases} \frac{x^2y^2}{x^4+y^4}, & (x,y) \neq (0,0); \\ 0, & (x,y) = (0,0). \end{cases}$$

(a) Show that $\lim\limits_{(x,y)\to(0,0)} g(x,y)$ does not exist.

(b) Show that the partial derivatives g_x and g_y both exist at $(0,0)$. What are their values at $(0,0)$?