**Math 114** **FINAL EXAM** **May 5, 2014**

**Circle one:** **Professor Pimsner**
**Professor Scherr**
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**Professor Ziller**

Name: ______________________________

Penn Id#: ____________________

Signature: ____________________________

TA: ____________________________

Recitation Day and Time: ____________________

You need to show your work, even for multiple choice problems. A correct answer with no work will get you 0 points. If you see a shortcut, you need to explain it. Please circle the answer for each multiple choice problem, and for all other problems put a square around the final answer. Each problem is worth 10 points. You are NOT allowed to use a calculator or cell phone, or any other electronic device.

(Do not fill these in; they are for grading purposes only.)

| | |
|---|---|
| 1) | 9) |
| 2) | 10) |
| 3) | 11) |
| 4) | 12) |
| 5) | 13) |
| 6) | 14) |
| 7) | 15) |
| 8) | |

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Total

**1.** Find the length of the curve given in parametric form by

$$\mathbf{r}(t) = 3\sin t\ \mathbf{i} + 3\cos t\ \mathbf{j} + 2t^{3/2}\mathbf{k}.$$

for $3 \le t \le 8$.

Answer:

(a) 30 (b) 38 (c) 25 (d) 19 (e) 27

**2.** Max is walking on a mountain whose height is described by $H(x, y) = e^{x/y}$. Presently he is located at the point $(2, 1)$. In what direction should he travel to get down the mountain as quickly as possible.

Answer:

(a) $\langle 1, -2\rangle$ (b) $\langle -1, 2\rangle$ (c) $\langle 0, 1\rangle$ (d) $\langle 2, -1\rangle$ (e) $\langle 1, 1\rangle$

**3.** Determine local minima, local maxima, and saddle points for the function $f(x,y) = 3x^2y + y^3 - 3x^2 - 3y^2$.

**4.** Find the maximum and minimum of the function $f(x, y) = e^{xy}$ in the region $x^2 + 4y^2 \leq 1$.

**5.** Let $z = xe^{xy}$ and $x = \ln(t),\ y = e^t$. What is $\frac{dz}{dt}$ at the point $(x, y) = (0, e)$.

Answer:

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

**6.** Evaluate the double integral

$$\int_0^4 \int_{\sqrt{y}}^2 3\sqrt{1+x^3}\,dxdy.$$

Answer:

(a) 1 (b) $\frac{1}{3}$ (c) $\frac{52}{3}$ (d) $\frac{26}{3}$ (e) 4

**7.** Write down an iterated triple integral in CYLINDRICAL coordinates that computes the volume of the region inside both the sphere $x^2 + y^2 + z^2 = 4$ and the cylinder $x^2 + (y-1)^2 = 1$. Do NOT carry out the actual integration.

**8.** Let $\mathbf{F}$ be the vector field $\mathbf{F} = \langle 5xy, 6yz, 2z \rangle$. Let $C$ be the path obtained by the intersection of the surfaces $x = z^2$ and $y = z$. Find the work done by $\mathbf{F}$ when traveling along $C$ from $(0,0,0)$ to $(1,1,1)$.

Answer:

(a) 15 (b) 5 (c) $\frac{13}{5}$ (d) $\frac{26}{5}$ (e) 4

**9.** Let **F** be the vector field

$$\mathbf{F} = \langle y^2 + 2xe^y + 1, 2xy + x^2e^y + 2y \rangle.$$

Compute the work integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $C$ is the path

$$\mathbf{r}(t) = \sin t \ \mathbf{i} + \cos t \ \mathbf{j}, \qquad 0 \leq t \leq \frac{\pi}{2}$$

**10.** Let $R$ be the region in the plane with vertices $(1,0),\ (2,1),\ (1,3),\ (0,2)$. Evaluate the integral

$$\iint_R (y-x)e^{2x+y}\, dA$$

.

**11.** Compute the area of the region inside the cardioid $r = 2 + 2\cos\theta$.

Answer:

(a) $2\pi$ (b) $8\pi$ (c) $4\pi$ (d) $2\pi - 4$ (e) $6\pi$

**12.** Let $S$ be the surface consisting of the portion of the plane $3x + y + 2z = 6$ in the first octant. Compute the flux of the vector field $\mathbf{F} = \langle 2x, 4z, y \rangle$ through $S$ in the direction away from the origin.

Answer:

(a) 12 (b) 36 (c) 72 (d) 80 (e) 70

**13.** Let $D$ be the region in 3 space given by $x^2 + y^2 + z^2 \leq 1$ , $x \geq 0, y \geq 0, z \geq 0$, and $S$ the boundary of $D$.

If $\mathbf{F}$ is the vector field $\mathbf{F} = \langle xy^2, yz^2, zx^2 \rangle$, compute the outward flux of $\mathbf{F}$ through $S$.

Answer:

(a) $\frac{4}{3}\pi$ (b) $\frac{\pi}{3}$ (c) $\pi$ (d) $\frac{\pi}{10}$ (e) $\frac{\pi}{5}$

**14.** Let $S$ be the part of the elliptic paraboloid $z = 5 - 4x^2 - y^2$ lying above the plane $z = 1$, oriented with normal vector pointing downward. Compute the flux of $\nabla \times \mathbf{F}$ across $S$, where $\mathbf{F}$ is the vector field $\mathbf{F} = \langle -yz, xz^2, xyz \rangle$.

**15.** A ball is thrown at ground level and after 5 seconds lands 10 meters away. What was the initial speed? The gravitational constant is 10 meters per second squared.

**Final Answers**

**1) 38**

**2)** $< -1, 2 >$

**3) local max at (0,0), local min at (0,2) and saddle at (1,1) and (-1,1)**

**4) min of** $e^{-1/4}$**, and max of** $e^{1/4}$

**5) 1**

**6) 52/3**

**7)** $\int_{-\pi/2}^{\pi/2} \int_0^{2\sin(\theta)} \int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} r \, dz \, dr \, d\theta$

**8) 5**

**9) 1**

**10)** $\frac{1}{2}(e^5 - e^2)$

**11)** $6\pi$

**12) 36**

**13)** $\pi/10$

**14)** $-4\pi$

**15)** $\sqrt{629}$