Math 114 **FINAL EXAM** **December 17, 2013**

Circle one: **Professor Krishnan**

Professor Shatz

Professor Yip

Professor Ziller

Name: ____________________________________

Penn Id#: ________________________

Signature: __________________________________

TA: __________________________________

Recitation Day and Time: ________________________

You need to show your work, even for multiple choice problems. A correct answer with no work will get you 0 points. If you see a shortcut, you need to explain it. Please circle the answer for each multiple choice problem, and for all other problems put a square around the final answer. Each problem is worth 10 points. You are NOT allowed to use a calculator or cell phone, or any other electronic device.

(Do not fill these in; they are for grading purposes only.)

1)	9)
2)	10)
3)	11)
4)	12)
5)	13)
6)	14)
7)	15)
8)	

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Total

1. A projectile is launched from the ground at an angle of $\frac{\pi}{4}$, and with an initial speed of $48\sqrt{2}$ feet/sec. How many seconds does it take a projectile to reach a height of 32 feet for the first time? Take the gravitational acceleration g to be 32 feet/sec^2.

Answer:

(a) 2 (b) 4 (c) 6 (d) 8 (e) 1 (f) 3

2. A curve C in 3-space is defined by

$$\mathbf{r}(t) = (4\cos t)\mathbf{i} + (4\sin t)\mathbf{j} + 3t\mathbf{k}.$$

Find the point p_0 on the curve C which has distance $\frac{5\pi}{4}$ from the point $(4, 0, 0)$, as measured along the curve.

3. A surface is described implicitly by $\ln \frac{y}{z} = e^{xy}$. Find the partial derivative $\frac{\partial z}{\partial y}$ at the point $(0, e, 1)$.

Answer:

(a) e (b) $1/e$ (c) $3e$ (d) $3/e$ (e) e^2 (f) -8

4. Let $f(x, y) = x^2y + \ln(xy)$. Answer the following for the derivatives at the point $(1, 1)$:

a) What is the derivative in the direction of $\mathbf{i} - \mathbf{j}$.

b) Find a direction (one is sufficient) in which the derivative is equal to 3.

c) Is there a direction in which the derivative is equal to 4? Justify your claim.

5. Let $f(x, y) = x^4 + y^4 - 4xy + 1$. Find all critical points and determine wether they are local maximum, local minimum or saddle points.

6. The maximum of the function $f(x, y) = e^{xy}$ on the disc $x^2 + y^2 \leq 1$ is equal to:

7. Evaluate the double integral

$$\int_0^4 \int_{\sqrt{y}}^2 \cos x^3 \, dx dy.$$

Answer:

(a) $\frac{1}{3}\sin(64)$ (b) $\sin(8)$ (c) $\cos(8) - 1$ (d) $\frac{1}{3}\sin(8)$ (e) $\frac{1}{3}\sin(2)$ (f) $\sin(2)$

8. A plate described by $1 \leq x^2 + y^2 \leq 9$ has mass density given by $\delta(x, y) = e^{x^2+y^2}$. What is the total mass of the plate?

Answer:

(a) $\frac{1}{8}(e^9 - e)$ (b) $\pi(e^9 - 1)$ (c) $\pi(e^9 - e)$ (d) $\frac{1}{8}(e^3 - e)$ (e) $\frac{1}{8}(e^3 - 1)$ (f) $e^9 - e$

9. Compute the volume of the solid bounded by the cone $z = 3\sqrt{x^2 + y^2}$, the plane $z = 0$, and the cylinder $x^2 + (y-1)^2 = 1$.

10. Evaluate the double integral

$$\iint_R \frac{e^{y+2x}}{y-x}\, dA$$

where R is the parallelogram with vertices $(0,2)$, $(1,3)$, $(0,5)$, $(-1,4)$.

11. Find the work done by the force field

$$\mathbf{F}(x,y) = e^y \sin(x)\,\mathbf{i} - (e^y \cos(x) - \sqrt{1+y})\,\mathbf{j}$$

in moving a particle from $(-\pi, \pi^2)$ to (π, π^2) along the parabola $y = x^2$.

Answer:

(a) π (b) e^π (c) 1 (d) $-\pi$ (e) $\sqrt{1+\pi}$ (f) 0

12. Use Green's theorem to evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where

$$\mathbf{F} = (e^{y^2} - 2y)\mathbf{i} + (2xye^{y^2} + \sin(y^2))\mathbf{j}$$

and C goes along a straight line from $(0,0)$ to $(1,2)$ and continues along a straight line to $(3,0)$.

13. Find the flux $\iint_S \mathbf{F}\cdot\mathbf{n}\ \mathbf{d}\sigma$ of the vector field $\mathbf{F} = y\mathbf{i}+x\mathbf{j}+z\mathbf{k}$ where the surface S is the sphere $x^2+y^2+z^2=1$ and $\mathbf{n}$ is the outward pointing unit normal.

Answer:

(a) $\frac{4}{3}$ (b) $\frac{4\pi}{3}$ (c) π (d) $\frac{\sqrt{\pi}}{16}$ (e) $\frac{\pi}{16}$

14. Let C be the curve that is the intersection of the plane $2x+z=1$ and the cylinder $(x-1)^2+y^2=9$ oriented counter-clockwise as viewed from above. Evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$ where

$$\mathbf{F}(x,y,z) = 10z\,\mathbf{i} + \sin(y^2)\,\mathbf{j} + e^{z^2}\,\mathbf{k}.$$

Answer:

(a) $-\pi$ (b) $e^2-\pi$ (c) 0 (d) π (e) $\sin(1)$

15. Find $|\mathbf{r}(1)|$ if $|\mathbf{r}(0)| = 0$ and $(\mathbf{r} \cdot \dot{\mathbf{r}})(t) = 6t^2$ for all t.

Answer:

(a) 0 (b) 4 (c) 6 (d) 27 (e) 54 (f) 2

Math 114 **FINAL EXAM Answers** **Fall 2013**

1) (e)

2)$r(\frac{5\pi}{4}) = 2\sqrt{2}\mathbf{i} + 2\sqrt{2}\mathbf{j} + \frac{3\pi}{4}\mathbf{k}$

3) (b)

4) (a) $\frac{1}{\sqrt{2}}$ (b) $\mathbf{v} = \mathbf{i}$ (c) No

5) (0,0) saddle point, (1,1) and (-1,-1) local minimum.

6) maximum is $\sqrt{e}$

7) (d)

8) (a)

9) $\frac{32}{3}$

10) $\frac{1}{3}(e^5 - e^2)\ln\frac{5}{2}$

11) (f)

12) -3

13)(b)

14) (c)

15) (f)