# Math 114, FINAL EXAM

## December 13, 2002

| INSTRUCTIONS | OFFICIAL USE ONLY |
|---|---|

**INSTRUCTIONS:**

1. Please complete the information requested below. There are 14 multiple choice problems and 6 open answer problems. No partial credit will be given on the multiple choice questions.
2. Please show all your work on the exam itself. Correct answers with little or no supporting work will not be given credit.
3. Only 18 of the 20 problems on the exam will count towards your final grade. We will automatically drop the worst score answer from each part (multiple choice and open answer) of the exam.
4. You are allowed to use one hand-written sheet of paper with formulas. *No calculators, books or other aids are allowed.* Please turn in your crib sheet together with your exam.

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• **Name** (please print):

• **Name of your professor:**
◯ **Dr. Hughes** ◯ **Dr. Pantev** ◯ **Dr. Preston**

• **Name of your TA:** ◯ **Ian Blumenfeld** ◯ **Christine Devena** ◯ **Madeeha Khalid** ◯ **Maria Sabitova** ◯ **Bill Semus**

• **I certify that all of the work on this test is my own.**
**Signature:**

• **Recitation section:**

| Problem | Points |
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| Total | |

# Part I: Multiple choice questions

**1.** Find the point at which the tangent line to the curve

$$x = 3t^2 - t \qquad y = 2t + t^3$$

at $t = 1$ intersects the line $y = 2 - x$.

(A) $\left(\frac{4}{5}, \frac{6}{5}\right)$ (B) $\left(\frac{1}{2}, \frac{3}{2}\right)$ (C) does not exist

(D) $(1, 1)$ (E) $\left(\frac{2}{3}, \frac{4}{3}\right)$ (F) $\left(\frac{3}{4}, \frac{5}{4}\right)$

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**2.** Find the length of the curve $r = \cos(\theta) - \sin(\theta)$, $0 \leq \theta \leq \frac{\pi}{4}$.

(A) $\frac{\pi}{2}$ (B) 1 (C) $\frac{1}{\sqrt{2}}$

(D) $\frac{\pi}{2\sqrt{2}}$ (E) $\frac{1}{4}$ (F) 2

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**3.** Which of the sets described by the cylindrical coordinate inequalities are unbounded? (Assume $r$ is always positive.)

(I) $r < 1$, $z < 1$
(II) $r + z^2 < 1$
(III) $z + r^2 < 1$

(A) (I) only (B) (II) only (C) (III) only

(D) (I) and (II) (E) (I) and (III) (F) (II) and (III)

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**4.** Evaluate the double integral

$$\int_0^1 \int_x^1 \sqrt{2+y^2}dydx.$$

(A) $\frac{2}{3}$ (B) $\sqrt{3}-\frac{2}{3}\sqrt{2}$ (C) $\frac{4}{3}+\sqrt{2}$

(D) 0 (E) $\frac{1}{3}$ (F) none of the above

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**5.** Which of these functions could be the general solution of a linear homogeneous second-order differential equation with constant coefficients?

(I) $y(x) = C_1 e^{C_2 x}$
(II) $y(x) = C_1 \sin x + C_2$
(III) $y(x) = C_1 x e^x + C_2 x e^{2x}$

(A) (I) only (B) (II) only (C) (III) only

(D) (II) and (III) (E) all of them (F) none of them

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**6.** Let $z$ be a complex number satisfying

$$\frac{1}{z} = \frac{2+i}{1-i}.$$

What is $\mathrm{Im}(z)$?

(A) $\sqrt{2}$ (B) $\sqrt{5}$ (C) $2i$

(D) $-\frac{3}{5}$ (E) $\frac{1}{5}i$ (F) $\frac{2}{5}$

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**7.** Compute

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} \sin\left(\pi[x^2+y^2]\right)\,dy\,dx$$

using polar coordinates.

(A) 0 (B) $\frac{\pi}{2}$ (C) $\pi$

(D) 1 (E) $\pi\sqrt{2}$ (F) $2\pi$

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**8.** Identify the functions which satisfy

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y}$$

(I) $f(x,y) = \sin x\cos y + \sin y\cos x$
(II) $f(x,y) = y\ln x + x\ln y$
(III) $f(x,y) = e^{xy}$

(A) (I) only (B) (II) only (C) (II) only(III) only

(D) (I) and (II) (E) (I) and (III) (F) (II) and (III)

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**9.** Where does the normal line to the surface $x^2+2y+2z=3$ at the point $(-1,0,1)$ intersect the $xy$-plane?

(A) $(1,1,0)$ (B) $(1,-1,0)$ (C) $(0,1,0)$

(D) $(0,-1,0)$ (E) $(-1,1,0)$ (F) $(-1,-1,0)$

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**10.** If $z = e^x\cos y$ and $x = \ln 2$ with an error of 0.1, and $y = \pi$ with an error of 0.2, what is the maximum error in $z$?

(A) 0 (B) 0.1 (C) 0.2 (D) 0.3 (E) 0.4 (F) 0.5

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**11.** Find the volume under the plane $z = 1 - x - y$, above the plane $z = 0$, and enclosed by the region $x \geq 0$, $y \geq 0$, $x + y \leq 1$.

(A) $\frac{1}{6}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{1}{3}$ (E) $\frac{1}{2}$ (F) 1

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**12.** Let $S$ be the surface given in cylindrical coordinates by the equation $z = 1 + r^2$. The intersection of $S$ with the plane $3y - \pi x = 0$ is

(A) empty (B) a hyperbola (C) a pair of lines

(D) a point (E) a parabola (F) a circle

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**13.** Find the absolute maximum of $f(x, y) = 9x^2y$ on the triangle bounded by the $x$-axis, the $y$-axis, and the line $x + y = 1$.

(A) 0 (B) 9 (C) 3 (D) $\frac{4}{3}$ (E) 3 (F) $\frac{9}{2}$

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**14.** A function $y(x)$ satisfies the intial value problem

$$xy' + y = 2x^2, \quad y(2) = \frac{1}{6}.$$

Find $y(1)$.

(A) $\frac{1}{6}$ (B) $-\frac{13}{3}$ (C) 0 (D) 1 (E) $-5$ (F) 11

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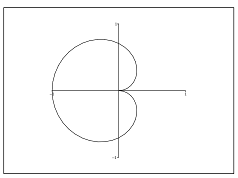
# Part II: Open answer questions

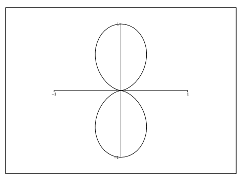
**15.** Match the polar coordinate equations to their graphs, for $0 \le \theta \le 2\pi$, $r \ge 0$. (**Hint:** it is easier to use the polar equations directly than to convert to Cartesian equations.) Enter your answers in the table below. Give a reason for each choice.

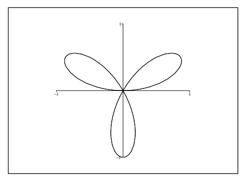
**Equations:**

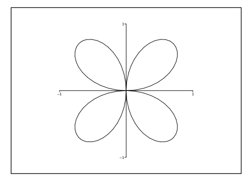
(i) $r = \sin^2(\theta/2)$
(ii) $r = \sin(3\theta)$
(iii) $r = \sin^2(\theta)$
(iv) $r = \sin^2(2\theta)$

**Graphs:**

(a) 

(b) 

(c) 

(d) 

**Answers:**

| **Graphs** | (a) | (b) | (c) | (d) |
|---|---|---|---|---|
| **Equations** | | | | |

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**16.** True or false. Explain your reasoning.

(a) If $|\vec{a} \times \vec{b}| = 3$, then $|(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})| = 6$.
(b) If $\widehat{\imath} \times \vec{a} = \vec{0}$ and $\widehat{\imath} \cdot \vec{a} = -1$, then $|\vec{a}| = 1$.

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**17.** Show that

$$\lim_{(x,y)\to(0,0)} \frac{\sin(x) - \sin(y)}{x + y}$$

does not exist.

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**18.** Let $f(x, y, z)$ be a function which at the point $(1, 0, 1)$ increases most rapidly in the direction of the vector $\widehat{\imath} + \widehat{\jmath} + \widehat{\mathbf{k}}$, with a rate of increase 2. Find the directional derivative of $f$ starting at the point $(1, 0, 1)$ and going in the direction parallel to the vector $\widehat{\imath} - \widehat{\jmath} + \widehat{\mathbf{k}}$.

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**19.** Show that $\rho(x) = x$ is an integrating factor of

$$(3x + 2y^2)\, dx + 2xy\, dy = 0$$

and then solve the equation using the integrating factor.

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**20.** Find the distance from the point $A = (1, 0, 2)$ to the plane passing through the point $(1, -2, 1)$ and perpendicular to the line given by the parametric equations $x = 7$, $y = 1 + 2t$, $z = t - 3$.

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