# Math 114, MAKEUP FINAL EXAM
## January 15, 2003

| INSTRUCTIONS | OFFICIAL USE ONLY |
|---|---|

**INSTRUCTIONS:**

1. Please complete the information requested below. There are 14 multiple choice problems and 6 open answer problems. No partial credit will be given on the multiple choice questions.
2. Please show all your work on the exam itself. Correct answers with little or no supporting work will not be given credit.
3. Only 18 of the 20 problems on the exam will count towards your final grade. We will automatically drop the worst score answer from each part (multiple choice and open answer) of the exam.
4. You are allowed to use one hand-written sheet of paper with formulas. *No calculators, books or other aids are allowed.* Please turn in your crib sheet together with your exam.

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• **Name** (please print):

• **Name of your professor:**
◯ **Dr. Hughes** ◯ **Dr. Pantev** ◯ **Dr. Preston**

• **Name of your TA:** ◯ **Ian Blumenfeld** ◯ **Christine Devena**
◯ **Madeeha Khalid** ◯ **Maria Sabitova** ◯ **Bill Semus**

• **I certify that all of the work on this test is my own.**
**Signature:**

• **Recitation section:**

| Problem | Points |
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| Total | |

# Part I: Multiple choice questions

**1.** What is the $x$-intercept of the line tangent to the curve $x(t) = 3+\cos(\pi t)$, $y(t) = t^2+t+1$ when $t = 1$?

(A) 3 (B) 2 (C) does not exist (D) 1 (E) $\frac{2}{3}$ (F) $\frac{5}{4}$

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**2.** Find the angle between the curves $y = x^2$ and $x = y^3$ at the point $(1, 1)$.

(A) 0 (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{3}$ (E) $\frac{\pi}{2}$ (F) undefined

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**3.** Find the area inside the lemniscate $r = \frac{2}{3}\cos(\theta)$, $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$.

(A) $\frac{4}{3}$ (B) $\frac{2\pi}{3}$ (C) $\frac{1}{3\sqrt{3}}$ (D) $\frac{2}{9}$ (E) $\frac{1}{\sqrt{3}}$ (F) $\frac{\pi}{9}$

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**4.** Find the integral that is equivalent to

$$\int_0^2 \int_{x^2}^{2x} f(x, y)\, dy\, dx$$

(A) $\int_0^2 \int_{y^2}^{2y} f(x, y)\, dx\, dy$ (B) $\int_0^4 \int_{y/2}^{\sqrt{y}} f(x, y)\, dx\, dy$ (C) $\int_0^1 \int_{\sqrt{y}}^{y/2} f(x, y)\, dx\, dy$

(D) $\int_0^2 \int_{2y}^{y^2} f(x, y)\, dx\, dy$ (E) $\int_0^2 \int_{y^2/2}^{\sqrt{2y}} f(x, y)\, dy\, dx$ (F) $\int_0^4 \int_{y}^{\sqrt{y/2}} f(x, y)\, dy\, dx$

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**5.** Use polar coordinates to evaluate the integral

$$\int\int_D \sqrt{x^2+y^2+3}\,dxdy,$$

where $D = \{(x,y) | x^2+y^2 \le 1, y \ge |x|\}$.

(A) $0$ (B) $-1$ (C) $\frac{\pi}{4}(9-\sqrt{2})$ (D) $\frac{\pi}{2}$ (E) $\frac{\pi}{6}(8-3\sqrt{3})$ (F) $\frac{2}{3}$

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**6.** Find the distance from the point $P = (1,1,1)$ to the line passing through $A = (1,0,-1)$ and perpendicular to the plane $x - y + z = 1$.

(A) $\sqrt{2}$ (B) $\sqrt{3}$ (C) $2$ (D) $\sqrt{\frac{14}{3}}$ (E) $\sqrt{\frac{11}{6}}$ (F) $\sqrt{7}$

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**7.** Let $z = e^x y + \cos(x)$ and let $x$ and $y$ be functions of $t$ satisfying $x(0) = 0$, $x'(0) = 3$ and $y(0) = 2$, $y'(0) = -1$. Find $z'(0)$.

(A) $5$ (B) $e$ (C) $0$ (D) $1/2$ (E) $-1$ (F) undefined

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**8.** Find the point of intersection of the line $x = 1 + 2t$, $y = 3t$, $z = t - 1$ and the plane tangent to $x^2 + 2y = z^2$ at $(2, 0, -2)$.

(A) $(3,3,0)$ (B) $\left(0, -\frac{3}{2}, -\frac{3}{2}\right)$ (C) $(1,0,-1)$

(D) $(-1,-3,-2)$ (E) $\left(\frac{1}{2}, \frac{1}{6}, -\frac{1}{2}\right)$ (F) $(7,9,2)$

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**9.** Use the linear approximation of the function $f(x,y) = \sin(\ln(1+x)y)$ at $(0,\pi)$ to estimate $f(-0.1, \pi)$.

(A) $-0.1\pi$ (B) $0.9\pi$ (C) $0$ (D) $0.35\pi$ (E) $0.3$ (F) $-0.2$

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**10.** Suppose $y(x)$ satisfies the differential equation

$$y' = x^2 + y^2$$

with initial condition $y(0) = -1$. Find $y''(0)$. **Hint:** you don't need to solve the differential equation.

(A) $-2$ (B) $-1$ (C) 0 (D) 1 (E) 2 (F) does not exist

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**11.** Which of these functions is continuous at $(0, 0)$?

(I) $f(x, y) = \dfrac{x^2 + y}{x^2 + y^2}$

(II) $f(x, y) = \dfrac{x + y}{x^2 + y^2}$

(III) $f(x, y) = \dfrac{xy^2}{x^2 + y^2}$

(A) (I) only (B) (II) only (C) (III) only

(D) (I) and (II) (E) (I) and (III) (F) (II) and (III)

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**12.** The function $f(x, y) = 8x^3 + y^3 + 6xy$ has

(A) one local maximum and one local minimum
(B) one saddle point and one local maximum
(C) one saddle point and one local minimum
(D) two saddle points
(E) two local maxima
(F) two local minima

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**13.** Which of these differential equations is exact as written?

(I) $(x + y^2)\,dx + (2xy + y^3)\,dy = 0$

(II) $\left(x + \dfrac{1}{y}\right) dx + \left(y + \dfrac{1}{x}\right) dy = 0$

(III) $\left(1 - \dfrac{y}{x^2}\right) dx + \left(y + \dfrac{1}{x}\right) dy = 0$

(A) (I) only (B) (II) only (C) (III) only

(D) (I) and (II) (E) (I) and (III) (F) (II) and (III)

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**14.** Find the difference of the absolute maximum and the absolute minimum of the function $f(x, y) = 7x - 4y + 3$ on the curve $(x - 1)^2 + (x - y)^2 = 25$.

(A) 1 (B) $-6$ (C) 50 (D) 12 (E) $-7$ (F) 4

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Part II begins on the next page

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# Part II: Open answer questions

**15.** True or false. Explain your reasoning.

(a) The surface given in cylindrical coordinates by the equation $r = \pi z \csc(\theta)$ is a plane.

(b) The spherical equations $\phi = \pi/3$, $|\theta| = \pi/4$ describe a single ray.

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**16.** Use polar coordinates to describe the region bounded from above by the unit circle centered at the origin and from below by the line $y = 1/2$. Give your answer in the form $a \le \theta \le b$, $f_1(\theta) \le r \le f_2(\theta)$ for some constants $a$, $b$ and some functions $f_1(\theta)$, $f_2(\theta)$.

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**17.** Find all the complex roots of the equation

$$z^4 + 4 = 0.$$

Give your answer in polar form.

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**18.** Solve the initial value problem:

$$x\frac{dy}{dx} + 3y = \frac{1}{x^2}, \qquad y(1) = 2.$$

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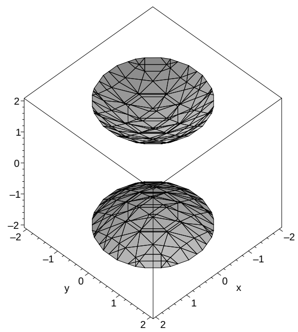
**19.** Match the equations to their graphs. Enter your answers in the table at the bottom of the page and show your reasoning on the next page.

**Equations:**

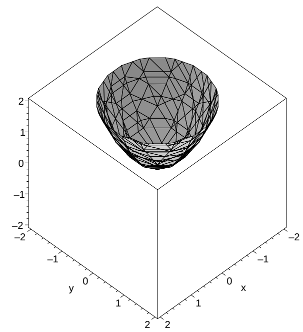
(1) $z - x^2 - y^2 = 0$ (2) $z^2 - x^2 - y^2 = 1$

(3) $\sin(x)y + \sin(y)x = 1$ (4) $z^2 - x^2 = 2$

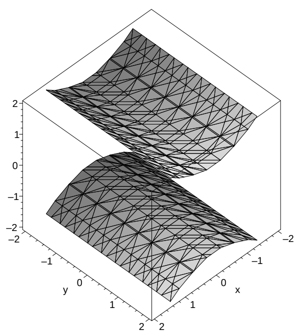
**Graphs:**



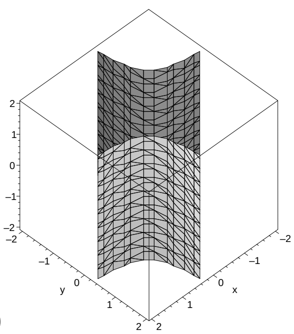
(a)



(b)



(c)



(d)

**Answers:**

| **Graphs** | (a) | (b) | (c) | (d) |
|---|---|---|---|---|
| **Equations** | | | | |

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**20.** Suppose a particular solution of

$$y'' + r(x)y' + s(x)y = x$$

is of the form $y_p(x) = Ax^3 + Bx^2$, where $A$ and $B$ are certain constants. Find the functions $r(x)$ and $s(x)$, the constants $A$ and $B$, and the general solution.