

5. The ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

bounds a region of volume $V = \frac{4}{3}\pi abc$. At a moment when $a = 1$, $b = 3$, and $c = 5$, a is changing at a rate of $+2$ and b is changing at a rate of -3 , at what rate must c be changing for the volume to remain constant?

- (A) -7 (B) -5 (C) -3 (D) -1
(E) 1 (F) 3 (G) 5 (H) 7
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6. The function $f(x, y) = x^3 - 6xy - 3y^2$ has

- (A) one local maximum and one saddle point
(B) one local minimum and one saddle point
(C) one local maximum and one local minimum
(D) two local maxima
(E) two local minima
(F) two saddle points
(G) a local maximum and no other critical points
(H) a saddle point and no other critical points
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7. Find the maximum value of the function $f(x, y, z) = xy + yz$ on the surface of the sphere $x^2 + y^2 + z^2 = 4$.

- (A) 0 (B) $\frac{1}{\sqrt{2}}$ (C) $\frac{1}{2\sqrt{2}}$ (D) $\frac{1}{2}$
(E) 1 (F) $\sqrt{2}$ (G) 2 (H) $2\sqrt{2}$
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8. Calculate $\iint_T x^2 dA$ where T is the triangular region with vertices $(0, 0)$, $(1, 0)$ and $(1, 2)$.

- (A) $1/3$ (B) $1/2$ (C) $2/3$ (D) 1
(E) $4/3$ (F) $3/2$ (G) $5/3$ (H) 2
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9. Calculate $\int_0^2 \int_{x^2}^4 \frac{e^{\sqrt{y}}}{y} dy dx$

- (A) e^2 (B) $2e^2$ (C) $2e^2 - 1$ (D) $2e^2 - 2$
(E) $e^{\sqrt{2}}$ (F) $2e^{\sqrt{2}}$ (G) $2e^{\sqrt{2}} - 1$ (H) $2e^{\sqrt{2}} - 2$

10. Find the volume of the solid bounded above by the paraboloid $z = 5 - x^2 - y^2$ and below by the paraboloid $z = 4x^2 + 4y^2$.

- (A) $\frac{\pi}{4}$ (B) 1 (C) $\frac{\pi}{2}$ (D) π
(E) $\frac{3\pi}{2}$ (F) 2π (G) $\frac{5\pi}{2}$ (H) $\frac{7\pi}{2}$
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11. Use the transformation $u = 3x + y$, $v = x + 2y$ to evaluate

$$\iint_R (3x^2 + 7xy + 2y^2) dA$$

where R is the region in the plane bounded by the lines $3x + y = 1$, $3x + y = 2$, $x + 2y = 0$ and $x + 2y = 1$.

- (A) $\frac{3}{20}$ (B) $\frac{3}{16}$ (C) $\frac{1}{8}$ (D) $\frac{1}{10}$
(E) $\frac{1}{2}$ (F) $\frac{3}{4}$ (G) $\frac{1}{4}$ (H) $\frac{5}{4}$
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12. Compute the volume of the solid bounded by the four surfaces $x + z = -1$, $x + z = 1$, $z = 1 - y^2$, and $z = y^2 - 1$.

- (A) $\frac{2}{3}$ (B) $\frac{5}{3}$ (C) $\frac{8}{3}$ (D) $\frac{11}{3}$
(E) 4 (F) $\frac{16}{3}$ (G) $\frac{20}{3}$ (H) $\frac{22}{3}$
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13. Compute the work done by the force field

$$\vec{F} = (6xy - y^3) \mathbf{i} + (3y^2 + 3x^2 - 3xy^2) \mathbf{j}$$

along the path $\mathbf{c}(t) = (\cos(t), \sin(t))$ for $-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$.

- (A) 0 (B) $\frac{1}{4}$ (C) 1 (D) -1
(E) $-\frac{1}{2}$ (F) $-\frac{1}{4}$ (G) -2 (H) 2

14. Evaluate

$$\int_C xy^3 dx + 3x^2y^2 dy$$

where C is the boundary of the region in the first quadrant enclosed by the x -axis, the line $x = 1$ and the curve $y = x^3$, traversed counter-clockwise.

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|-------------------|--------------------|-------------------|-------------------|
| (A) 0 | (B) $\frac{1}{2}$ | (C) $\frac{1}{3}$ | (D) $\frac{1}{4}$ |
| (E) $\frac{1}{6}$ | (F) $\frac{1}{11}$ | (G) 1 | (H) 2 |
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15. Let $y(t)$ be the solution to the differential equation

$$t \frac{dy}{dt} = t^2 + 2y$$

satisfying $y(1) = 2$. What is $y(2)$?

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|-------------------|-------------------|-------------------|-------------------|
| (A) $2 + 4 \ln 2$ | (B) $8 + 4 \ln 2$ | (C) $4 + 2 \ln 2$ | (D) $8 + 8 \ln 2$ |
| (E) $2 + 8 \ln 2$ | (F) $4 + 4 \ln 2$ | (G) $2 + 2 \ln 2$ | (H) $4 + 8 \ln 2$ |