Math 114 - Final Exam - University of Pennsylvania

Tuesday, May 4, 2010 @ 12:00 - 2:00 PM

- **Name**: ______________________________
- **Your professor (circle one)**: Dr. Powers Dr. He Dr. Wylie Dr. Strain
- **Your TA(circle one)** : Yan Wang Sohrab Shahshahani Zhe Liu Eugene So Ryan Eberhart Yin Xia
- **Recitation day and time**: ______________________________

I certify that all of the work on this test is my own.

Signature: ______________________________

INSTRUCTIONS:

1. Please complete the information requested above. There are 19 multiple choice problems.
2. Please show all your work on the exam itself. Correct answers with little or no supporting work will not be given credit.
3. You are allowed one hand-written sheet of paper with formulas. *No calculators, books or other aids are allowed.* Please turn in your crib sheet together with your exam.

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Problem	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Points	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Your score																			

Your total score: ______________________________

1. Let P be the plane that contains the points $(2,1,3)$, $(2,2,4)$, and $(1,1,6)$. What is the distance from P to the point $(1,1,1)$?

   (a) $\frac{5}{\sqrt{11}}$ **
   (b) $\frac{5}{3}$
   (c) $\frac{5}{\sqrt{13}}$
   (d) $\frac{8}{\sqrt{11}}$
   (e) $\frac{8}{3}$
   (f) $\frac{8}{\sqrt{13}}$
   (g) $\frac{13}{\sqrt{11}}$
   (h) $\frac{11}{\sqrt{13}}$

2. Where does the the plane that contains both of the lines

$$L_1 =< 3-t, -4+t, 4+2t > \quad \& \quad L_2 =< 3+t, -4+t, 4-t >,$$

   intersect the x-axis?

   (a) $x = 4$
   (b) $x = 6$
   (c) $x = 3$
   (d) $x = 7$ **
   (e) $x = -4$
   (f) $x = -6$
   (g) $x = -3$
   (h) $x = -7$

3. Consider the surface $z = x^2 + 2y^2 - 2x + 4y$. At one point $\langle x_0, y_0, z_0 \rangle$ the tangent plane to the surface is parallel to the xy-plane. What is the z-coordintate of that point?

   (a) -3**
   (b) -1
   (c) 0
   (d) 4
   (e) 6
   (f) 7
   (g) 9
   (h) 10

4. Find the maximum of the function $f(x, y, z) = 2x + 3y - 2z$ in the region

$$4x^2 + 6y^2 + 2z^2 \leq 18.$$

   (a) 0
   (b) 1
   (c) 3
   (d) 4
   (e) 6
   (f) 9 **
   (g) 12
   (h) 20

5. The function $y(t)$ satisfies the equation $\frac{dy}{dt} + 2ty = y$ and $y(0) = 5$. What is $y(1/2)$?

(a) 5
(b) 0
(c) $e^{1/4}$
(d) $5e^{1/4}$ **
(e) $-5e^{1/4}$
(f) $-e^{1/4}$
(g) -5
(h) e

6. Jack and Jill plan to evaluate the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r},$$

along a path in the xy-plane from $(0,0)$ to $(1,1)$. The force field is

$$\mathbf{F}(x,y) = (x + 2y, -x + y^2).$$

Jack chooses the path that runs along the x-axis from $(0,0)$ to $(1,0)$ and then runs along the vertical line $x = 1$ from $(1,0)$ to the final point $(1,1)$. Jill chooses the direct path along the diagonal line $y = x$ from $(0,0)$ to $(1,1)$. Whose line integral is larger and by how much?

(a) Jack's by 5/2
(b) Jack's by 2
(c) Jack's by 1
(d) They are equal.
(e) Jill's by 1
(f) Jill's by 3/2 **
(g) Jill's by 4
(h) Jill's by 9/2

7. Find the integral of the function $F(x, y, z) = z$ in the region E consisting of the top half of a sphere of radius 2 divided by the volume of that region which is $V = 16\pi/3$ (i.e. the region E is $z \geq 0$ and $x^2 + y^2 + z^2 \leq 4$):

$$\text{Compute:} \qquad \overline{z} = \frac{1}{V} \iiint_E z \, dV.$$

   (a) $1/3$
   (b) $1/2$
   (c) $1/\sqrt{2}$
   (d) $3/4$ **
   (e) $\pi/4$
   (f) $\sqrt{2}$
   (g) $\pi/2$
   (h) $3/\pi$

8. Calculate the following integral

$$\int_2^4 \int_{\frac{y-2}{2}}^1 e^{(x^2)} dx dy.$$

(You may need to change the order of integration.)

   (a) $2e$
   (b) $e - 1$ **
   (c) $1 - \frac{1}{e}$
   (d) $\sqrt{e} - 1$
   (e) $1 - \frac{1}{\sqrt{e}}$
   (f) $2e^2 - 2$
   (g) $2\sqrt{2}$
   (h) $4e - 2$

9. Use a double integral to find the area of the region enclosed by the curve

$$r = \sqrt{\cos(\theta)},$$

when $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$.

(a) $\frac{1}{2}$
(b) $\frac{46\pi}{27}$
(c) $\frac{\pi}{4}$
(d) $\frac{\pi}{8}$
(e) $\frac{\pi}{16}$
(f) 1 **
(g) 46π
(h) 4π

10. The electrical potential V is given by

$$V(x, y, z) = x^2 - y^2 - z^2.$$

In which direction does the potential increase most rapidly at $(1, 1, 0)$?

(a) $(1, 1, 0)$
(b) $(-2, 2, 0)$
(c) $(2, 2, 0)$
(d) $(-2, -2, 0)$
(e) $(1, 0, 0)$
(f) $(1, 1, 0)$
(g) $(1, -1, 0)$ **
(h) $(1, 1, 1)$

11. Consider the function $f(x, y) = -2x^3 + 3x^2 + 2y^2 - 4y$. Find the two critical points and determine their type.

(a) local min at $(x, y) = (0, 1)$,
local min at $(x, y) = (1, 1)$.

(b) local min at $(0, 1)$,
saddle at $(1, 1)$. **

(c) local min at $(0, 1)$,
local max at $(1, 1)$.

(d) saddle at $(0, 1)$,
local min at $(1, 1)$.

(e) saddle at $(0, 1)$,
saddle at $(1, 1)$.

(f) saddle at $(0, 1)$,
local max at $(1, 1)$.

(g) local max at $(0, 1)$,
local min at $(1, 1)$.

(h) local max at $(0, 1)$,
saddle at $(1, 1)$.

12. Compute the work done by the force

$$\mathbf{F}(x,y,z) =< 2x, 3y, -z >$$

along the path

$$\mathbf{r}(t) =< t, t^2, t^3 > \qquad 0 \le t \le 1.$$

Hint: This is given by the line integral

$$W = \int_C \mathbf{F} \cdot d\mathbf{r}$$

(a) -2
(b) -1
(c) 0
(d) 1
(e) 2 **
(f) 3
(g) 4
(h) 5

13. Compute the integral

$$\iint_R \cos\left(\frac{x-y}{x+y}\right) dA$$

where R is the region inside the triangle with vertices $(0,0)$, $(0,1)$, and $(1,0)$.

(a) $-\sqrt{2}$
(b) $\frac{\pi}{4}$
(c) $1 - \cos(1)$
(d) $1 - \frac{1}{\sqrt{2}}$
(e) $\frac{\pi}{2}$
(f) $\frac{\sin(1)}{2}$ **
(g) 0
(h) $\sin(1) - \cos(1)$

14. Suppose a tennis canon located at ground level fires a ball at a 45° angle to the ground with a speed of 80 feet per second. In calculating the trajectory of the ball you may exclude the effect of air resistance and assume the only force acting on the ball is gravity which accelerates the ball downward at a rate of 32 feet per second. How high is the ball at its highest point?

    (a) 20 feet
    (b) $20\sqrt{2}$ feet
    (c) 32 feet
    (d) 50 feet **
    (e) $50\sqrt{2}$ feet
    (f) 64 feet
    (g) $64\sqrt{2}$ feet
    (h) 100 feet

15. Let $\kappa(x)$ be the curvature of the parabola $y = x^2$ at (x, x^2). What is maximal value of $\kappa(x)$?

    (a) 1/2
    (b) 2 **
    (c) 1
    (d) 4
    (e) 1/4
    (f) 3
    (g) 1/3
    (h) 0

16. A helix has the vector equation $\mathbf{r}(t) = (\cos t, \sin t, 3t)$. Find the arc length from $\mathbf{r}(0)$ to $\mathbf{r}(2\pi)$.

(a) 2π
(b) 4π
(c) $2\sqrt{10}\pi$ **
(d) $2\sqrt{2}\pi$
(e) $\sqrt{10}\pi$
(f) $\sqrt{2}\pi$
(g) $4\sqrt{2}\pi$
(h) 20π

17. What is the volume of the region above the plane $z = 2$ and inside the sphere $x^2+y^2+z^2 = 9$. (i.e. $z \geq 2$ and $x^2 + y^2 = z^2 \leq 9$) ?

(a) 2π
(b) $\frac{8\pi}{3}$ **
(c) $\frac{5\pi}{2}$
(d) 4π
(e) $\frac{9\pi}{4}$
(f) $\frac{11\pi}{3}$
(g) 7π

18. Evaluate

$$\iint_D x \, dA$$

where D is the region in the xy-plane bounded by the curves $y = 2x$ and $y = x^2$.

(a) -1
(b) $-\frac{1}{2}$
(c) 0
(d) $\frac{3}{8}$
(e) $\frac{3}{4}$
(f) 1
(g) $\frac{4}{3}$ **
(h) $\frac{5}{2}$

19. Evaluate the following line integral.

$$\int_C (6y + x)dx + (y + 2x)dy$$

C is the circle $(x-2)^2 + (y-3)^2 = 4$ oriented counterclockwise.

(a) 16π
(b) 8π
(c) 32π
(d) 4π
(e) -16π **
(f) -8π
(g) -32π
(h) -4π