MATH 114, FINAL EXAM
MAY 7^{th} 2009, 12-2PM

Name: ________________________ **Penn ID number:** __________

Signature: ________________________

Circle the name of your instructor:

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Instructions:

1. The exam is 2 hours long and consists of 19 questions. Each question is worth 5 points.

2. You may use one handwritten two-sided page of notes. No other notes, books or calculators are allowed.

3. **You must show all work**; answers without supporting work will be given little or no credit.

The following is for grading purposes only, do not fill in.

1.	8.	15.
2.	9.	16.
3.	10.	17.
4.	11.	18.
5.	12.	19.
6.	13.	
7.	14.	

Total: __________

(1) Where does the plane through the points $(x, y, z) = (1, 0, 0)$, $(0, 1, 0)$, $(1, 1, 2)$ intersect the z-axis?
Answer: $z =$

A. -4 B. -2 C. -1 D. 0 E. 1 F. 2 G. 4 H. 6

(2) Suppose that $y(x)$ satisfies the differential equation $xy' + y = 2x$. Given the initial condition $y(1) = 4$, what is $y(2)$?

A. 4 B. 7/2 C. $2\sqrt{2}$ D. $3\sqrt{2}$ E. $\sqrt{3}$ F. 3 G. 5/2 H. 1

(3) Find the maximum of the function $F(x, y, z) = 2x + y - z$ on the surface

$$4x^2 + 2y^2 + z^2 = 40.$$

Max =

A. 1 B. 2 C. 4 D. 5 E. 7 F. 10 G. 13 H. 25

(4) Find the distance between the plane through the points $(x, y, z) = (1, 0, 0)$, $(0, 1, 0)$, $(0, 0, -2)$, and the origin $(0, 0, 0)$?
Distance =

A. $\frac{1}{2}$ B. $\frac{2}{3}$ C. $\frac{\sqrt{2}}{2}$ D. $\frac{4}{5}$ E. $\frac{6}{7}$ F. 1 G. $\frac{3}{2}$ H. 2

(5) Evaluate the integral

$$\iint_R x + y \, dA$$

where R is the the region inside the triangle with vertices $(x, y) = (0, 0)$, $(2, 0)$ and $(0, 1)$.

A. 0 B. 1/4 C. 1/3 D. 1/2 E. 2/3 F. 3/4 G. 1 H. 4/3

(6) The helix given by the parametric equations

$$x = 4\sin(t) \quad y = 4\cos(t) \quad z = 3t$$

has constant curvature. What is the curvature of this helix?

A. 0 B. $\frac{1}{17}$ C. $\frac{4}{25}$ D. $\frac{1}{4}$ E. $\frac{1}{3}$ F. $\frac{1}{2}$ G. $\frac{1}{\sqrt{2}}$ H. 1

(7) Find the volume of the region that is inside the sphere $x^2 + y^2 + z^2 = 4$ and above the cone $z = \sqrt{x^2 + y^2}$ (i.e., $z \geq \sqrt{x^2 + y^2}$ and $x^2 + y^2 + z^2 \leq 4$).

A. $8\pi/3$ B. $\frac{16\pi}{3}(1 - 1/\sqrt{2})$ C. $2\pi(\sqrt{2} - 1)$ D. $\frac{32\pi}{3}$

E. $\frac{16\pi}{5}$ F. 12π G. $4\pi(1 - 1/\sqrt{2})$ H. 18π

(8) Which of the following unit vectors points in the direction of fastest increase for the function $f(x, y) = (x^2 + y^2)e^{-xy}$ at the point $(1, 0)$?

A. $\langle 1/\sqrt{2}, 1/\sqrt{2}\rangle$ B. $\langle -3/5, 4/5\rangle$ C. $\langle 5/\sqrt{34}, 3/\sqrt{34}\rangle$ D. $\langle 0, -1\rangle$

E. $\langle 2/\sqrt{5}, -1/\sqrt{5}\rangle$ F. $\langle 1/2, \sqrt{3}/2\rangle$ G. $\langle -12/13, 5/13\rangle$ H. $\langle 1, 0\rangle$

(9) Evaluate the line integral

$$\oint_C -y\,dx + x\,dy,$$

where C is the closed curve running counterclockwise around the circle $x^2 + y^2 = 4$.

A. -5π B. -2π C. $-\pi$ D. 0 E. π F. 3π G. 8π H. 11π

(10) Suppose y is a function of t satisfying the differential equation $\dfrac{dy}{dt} = ky$, where k is a constant. Suppose y satisfies the initial conditions $y(0) = 4$ and $y(1) = 2$. What is $y(2)$?

A. 4 B. $4/e$ C. $2/e$ D. $\sqrt{2}$ E. $\sqrt{5}$ F. 1 G. $1/e$ H. 0

(11) Find the arc length of the plane curve $y = 2x^{3/2}$ from $x = 0$ to $x = 1/3$.

A. 1/4 B. 1/3 C. 15/64 D. 14/27 E. 11/167 F. 1/2 G. 4/3 H. $\sqrt{2}$

(12) The tangent plane to the surface $4x^2 + 2y^2 + z^2 = 10$ at $(x, y, z) = (1, 1, 2)$ intersects the z-axis at a unique point $(0, 0, z_0)$. What is z_0?

A. -1 B. 0 C. 1 D. 2 E. 4 F. 5 G. 7 H. 10

(13) A particle travels along a path C given by the vector-valued function

$$\mathbf{r}(t) = t\sqrt{\pi/2}\,\mathbf{i} + t^2(1 - t^2)\,\mathbf{j}$$

with $0 \le t \le 1$. As the particle moves along C it is subjected to a force given by the vector field

$$\mathbf{F}(x, y) = 2x\cos(x^2 + y^2)\,\mathbf{i} + (2y\cos(x^2 + y^2) + 1)\,\mathbf{j}.$$

Find the work done on the particle by the force.
(Recall that the work is given by the line integral $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r}$.)

A. $-e^{4\pi}$ B. 0 C. 1 D. $\dfrac{\pi^2}{4}$ E. 30 F. π G. -42 H. $\ln(10)$

(14) Evaluate the integral

$$\int_0^2 \int_{2y}^4 e^{-x^2/2}\,dx\,dy.$$

(You may need to change the order of integration)

A. $1 - e^{-1}$ B. $(1 - e^{-8})/2$ C. $2(1 - e^{-8})$ D. $1 - e^{-4}$

E. 2 F. $(1 - e^{-16})/4$ G. $e^2 - 1$ H. e^{-4}

(15) The space curves define by the vector-valued functions

$$\mathbf{r}(t) = (t^2, \sin(t), t^4) \quad \text{and} \quad \mathbf{s}(t) = (t^3, t, \sin(t))$$

intersect at the point $(0, 0, 0)$ when $t = 0$. What is the angle (in radians) between the two curves at the point $(0, 0, 0)$.

A. 0 B. $\pi/6$ C. $\pi/4$ D. $\pi/3$ E. $\pi/2$ F. $2\pi/3$ G. $3\pi/4$ H. $5\pi/6$

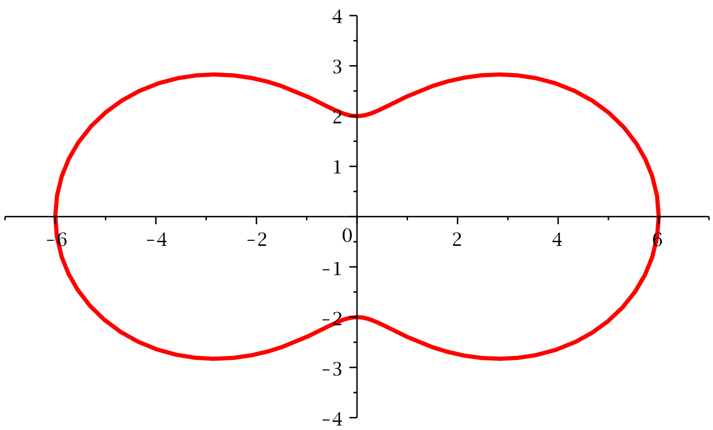
(16) A cylinder of solid metal is given by the region in space bounded by $x^2 + y^2 = 25$ and the planes $z = 0$ and $z = 4$. The density function of the cylinder is $\rho(x, y, z) = e^{x^2+y^2}$. What is the mass of the cylinder?

A. $4\pi(e^{25} - 1)$ B. 8π C. $8\pi(e^5 - 1)$ D. 10π

E. $10\pi e^{16}$ F. $\pi(e^{10} - 1)$ G. $\pi(e^{-25} - 1)/4$ H. 0

(17) What is the area of the region in the plane bounded by the curve given in polar coordinates by: $r = 4 + 2\cos(2\theta)$.

A. 2 B. $\sqrt{5}\pi$ C. 4π D. 16 E. 18π F. 11π G. 32 H. 14π



(18) The function $f(x, y) = x^4 + y^4 - 4xy + 1$ has how many local minima?

A. 0 B. 1 C. 2 D. 3 E. 4 F. 5 G. 6 H. 7

(19) In the sawdust mill, a log rolls down the conveyor belt and is power sanded from all sides. The log is shaped like a cylinder. When the length of the log is 10 feet and the radius is 2 feet, the length is decreasing at a rate of 3 feet/minute and the radius is decreasing at a rate of 1 feet/minute. What is the rate of change of the surface area at that time (in feet^2/minute)?

(Recall that the surface area of a cylinder is $2\pi r^2 + 2\pi rl$ where r is the radius and l is the length of the cylinder).

A. -10π B. -20π C. -30π D. -40π E. -50π F. -60π G. -70π H. -80π

Answer key: BBFBGCBEGFDFCBCAECD