Math 114: Final Exam

May 9, 2008

Instructions:

1. Please sign your name and indicate the name of your instructor and your teaching assistant:

   A. Your Name:

   B. Your Instructor:

   C. Your Teaching Assistant:

2. This exam is 2 hours long and there are 20 questions.
3. You can use one handwritten two-sided page of notes; no books or calculators are allowed.
4. **It is important that you show your work for each problem.** To receive credit for a problem, you must both indicate the correct answer and show plausible work justifying your answer.
5. Do not come to the front of the class when the exam is over; we will pick up your exam from you.
6. **Don't take any work with you which is needed to justify your answers.**
7. If you want to find out how you did on the exam, record you answers on the last page of the exam, tear this off and take it with you. We will post an answer key on the web.

The table below is for grading purposes only. You do not need to transfer your answers to this page.

1.	2.	3.	4.	5.
6.	7.	8.	9.	10.
11.	12.	13.	14.	15.
16.	17.	18.	19.	20.

Total:

(1) Let B be the box in $\mathbb{R}^3$ of all points (x, y, z) such that $0 \le x \le 1$, $0 \le y \le 1$ and $0 \le z \le 1$. Suppose that the probability density function that a fly will be at position (x, y, z) is

$$f(x, y, z) = y + z \quad \text{if} \quad (x, y, z) \in B \quad \text{and} \quad f(x, y, z) = 0 \quad \text{if} \quad (x, y, z) \notin B$$

What is the probability that the y coordinate of the fly will be between 0 and $1/2$?

(A) $3/8$	(B) $1/2$
(C) $5/8$	(D) $1/3$
(E) $2/3$	(F) none of the above

Answer to 1:

(2) Find the solution $y = y(x)$ the differential equation $\dfrac{dy}{dx} = 5x^4 y$ for which $y(0) = 2$.

(A) $y(x) = 3e^{x^5}$

(B) $y(x) = 2e^{x^4}$

(C) $y(x) = 3e^{x^4} + c$

(D) $y(x) = e^{x^5} + c$

(E) $y(x) = 2e^{x^5}$

(F) none of the above

Answer to 2:

(3) Consider the following three limits:

(I) $$\lim_{(x,y)\to(0,0)} \frac{x+y}{x^2+y^2}.$$

(II) $$\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}.$$

(III) $$\lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{x^2+y^2}.$$

Which of the following statements is true? (Select one answer.)

(A) All three limits converge

(B) None of the limits converge

(C) Only limit (I) converges

(D) Only limit (II) converges

(E) Only limit (III) converges

(F) Limits (I) and (II) converge, but limit (III) does not converge

Answer to 3:

(4) Which of the following vectors is normal to the plane determined by the points $A(2,1,0)$, $B(0,1,1)$, $C(-2,0,-1)$?

(A) $< 1,0,4 >$

(B) $< 0,1,0 >$

(C) $< 1,-1,4 >$

(D) $< 1,3,-2 >$

(E) $< 1,6,2 >$

(F) $< 1,-6,2 >$

(G) none of the above

Answer to 4:

(5) Evaluate the integral

$$\int_{y=0}^{y=8} \int_{x=y^{1/3}}^{2} e^{x^4} dx dy$$

by changing the order of integration.

(A) e^2

(B) $(e^2 - 1)/4$

(C) $(e^{16} - 1)$

(D) e^{16}

(E) $(e^{16} - 1)/4$

(F) none of the above

Answer to 5:

(6) Solve

$$(6+t)\frac{du}{dt} + u = 6 + t$$

(A) $u = \dfrac{t^2+6t}{t+6} + C$

(B) $u = \dfrac{t^3+6t}{t+6} + C$

(C) $u = \dfrac{t^2+6t+C}{t+6}$

(D) $u = \dfrac{t^2/2+6t+C}{t+6}$

(E) $u = \dfrac{-t^2/2+6t+C}{t-6}$

(F) None of the above.

Answer to 6:

(7) Let f be the function

$$f(x, y) = \ln(x + y)$$

for $(x, y) \in \mathbb{R}^2$ and $x + y > 0$. A unit vector in $\mathbb{R}^2$ is a vector u of length $|u| = 1$. What is the maximum value of the directional derivative $D_u(f)$ of f at the point $(x, y) = (2, -1)$ as u ranges over all unit vectors in $\mathbb{R}^2$?

(A) 1

(B) 1/2

(C) $\sqrt{2}$

(D) $\sqrt{3}$

(E) $\ln(2)$

(F) 0

(G) none of the above

Answer to 7:

(8) For each of the following statements, mark whether they are true or false. Each correct answer is worth two points. **No work needs to be shown for this problem.**

(a) For any 3-dimensional vectors $\mathbf{u}$ and $\mathbf{v}$, $\mathbf{u} \times \mathbf{v} = \mathbf{v} \times \mathbf{u}$. True◯ False◯

(b) The cross product of two unit vectors is a unit vector. True◯ False◯

(c) The vectors $< -1, 0, 1 >$ and $< -2, 0, 2 >$ are parallel. True◯ False◯

(d) The velocity vector of a curve in three dimensions is always perpendicular to the acceleration vector. True◯ False◯

(e) Two planes are parallel if the cross product of their normal vectors is 0. True◯ False◯

(9) Find the volume of the solid which lies both within the cylinder $x^2 + y^2 = 1$ and the sphere $x^2 + y^2 + z^2 = 4$.

(A) $\frac{2\pi}{3}(2 - \sqrt{3})$.

(B) $\frac{\pi}{3}(2 - \sqrt{3})$.

(C) $\pi(2 - \sqrt{3})$.

(D) $\frac{2}{3}(2 - \sqrt{3})$.

(E) $\frac{4\pi}{3}(8 - 3^{3/2})$

(F) none of the above

Answer to 9:

(10) Find a particular solution $y = y(x)$ to $y''(x) + y'(x) = \sin(x)$.

(A) $\sin(x)$

(B) $-\frac{1}{2}(\sin(x) + \cos(x))$

(C) $\cos(x)$

(D) $\frac{1}{2}(\sin(x) - \cos(x))$

(E) $-\cos(x)$

(F) none of the above

Answer to 10:

(11) Analyze the critical points of the function

$$f(x, y) = x^3 + x^2 - 2xy + y^2 - 3x + 1$$

Which of the following statements is true?

(A) The function has one local minimum and one saddle point

(B) The function has two local minima and one local maximum

(C) The function has one local maximum and one local minimum

(D) The function has three saddle points

(E) The function has one local maximum and one saddle point

(F) The function has one local minimum and one critical point where the second derivative test is inconclusive

(G) None of the above

Answer to 11:

(12) At $t = 0$ an airplane takes off. At that moment, its position vector is $< 0, 0, 0 >$, and its velocity vector is $< 1, 2, 0 >$. Find its position vector at time $t = 6$, if the acceleration of the airplane is $a(t) =< 1, 0, t >$.

(A) $< 20, 30, 40 >$ (B) $< 12, 16, 28 >$ (C) $< 10, 12, 30 >$

(D) $< 30, 20, 40 >$ (E) $< 24, 12, 36 >$ (F) $< 12, 6, 18 >$

(G) none of the above

Answer to 12:

(13) Find the surface area of the part of the plane $z = 2 + 3x + 4y$ which lies above the rectangle of points (x, y) in the x-y plane for which $0 \leq x \leq 5$ and $1 \leq y \leq 4$.

(A) $12\sqrt{26}$

(B) 75

(C) $15\sqrt{26}$

(D) 60

(E) none of above

Answer to 13:

(14) The first three non-zero terms in the power series solution of

$$y'' - xy' - y = 0$$

subject to $y(0) = 0$ and $y'(0) = 1$ are:

(A) $x - x^2 + x^3$

(B) $x + \frac{x^2}{2} + \frac{x^3}{3}$

(C) $x + \frac{x^3}{3} + \frac{x^5}{5}$

(D) $x + \frac{x^3}{3} + \frac{x^5}{15}$

(E) $x - \frac{x^3}{6} + \frac{x^5}{120}$

(F) none of the above

Answer to 14:

(15) The length ℓ, width w and height h of a box change with time t. At time $t = t_0$, the value of ℓ is 1 meter, the value of w is 2 meters and the value of h is 2 meters. Suppose that at time $t = t_0$, both ℓ and w are increasing at the rate of 2 meters per second, while h is decreasing at the rate of 3 meters per second. What is the rate of change in cubic meters per second of the volume V of the box at time $t = t_0$?

(A) 6	(B) 4	(C) 3
(D) -2	(E) 0	(F) none of the above

Answer to 15:

(16) The perpendicular from the point $P(1, 2, 1)$ to the plane $x - y + z = 1$ intersects that plane at the following point:

(A) $(1, 1, 1)$ (B) $(0, -1, 0)$ (C) $(1/3, -1/3, 1/3)$

(D) $(2/3, 1/3, 2/3)$ (E) $(4/3, 5/3, 4/3)$ (F) $(2, 3, 2)$

Answer to 16:

(17) A politician is planning his first visit to Bagdad. He expects the local citizens will want to shower his motorcade with rose petals. The secret service tells him it would be safer if he brought along enough rose petals to give the citizens for this purpose. The politician expects the citizens will want to cover his motorcade with a circular pattern of rose petals having mass density function given by

$$f(x,y) = 10 - \sqrt{(x^2+y^2)} \quad \text{for} \quad \sqrt{x^2+y^2} \leq 10$$
$$\text{and} \quad f(x,y) = 0 \quad \text{for all other} \quad (x,y).$$

What is the total mass of rose petals the politician should bring along with him?

(A) 5000π

(B) 1000π

(C) $2500\pi/3$

(D) $1000\pi/3$

(E) none of above

Answer to 17:

(18) Suppose that $y(t)$ is a solution of the initial value problem

$$\frac{dy}{dt} = y(1 - 0.0005y) \quad \text{and} \quad y(0) = 1.$$

What is the limit $\lim_{t \to +\infty} y(t)$?

(A) 1π

(B) 0.0005

(C) 2000

(D) 0

(E) ∞

(F) none of the above

Answer to 18:

(19) Find the equation of the tangent plane to the surface

$$4x^4 + 2y^4 + z^4 = 22$$

at the point $(1, 1, 2)$.

(A) $4x + 2y + z = 8$ (B) $2x + y + z = 5$ (C) $2x + y + 4z = 11$

(D) $2x + 2y + z = 7$ (E) $x + y + 4z = 10$ (F) $x + 2y + 4z = 11$

(G) none of the above

Answer to 19:

(20) Find the length of the curve parameterized by the function

$$\mathbf{r}(t) =< t^2/2, \sqrt{2}t - 2, \ln t >$$

for $1 \leq t \leq 2$.

(A) $3/2$ (B) $1/2 + 2\ln 2$ (C) $3/2 + \ln 3$

(D) 3 (E) $1/2 + \ln 3$ (F) $3/2 + \ln 2$

(G) none of the above

Answer to 20:

Math 114: Final Exam Answer Record

Answer Record Instructions:

(1) **Record your answers on this sheet.**

(2) **Tear off this sheet and take it with you after the exam.**

(3) **Check your answers against the key to be posted on your courses web page.**

Answers

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