UNIVERSITY *of* PENNSYLVANIA
Mathematics Department
MATH 114
FINAL EXAMINATION
SPRING 2006

NAME: ____

YOUR PROFESSOR (CHECK ONE): ❑CROTTY ❑DETURCK ❑KOENIGSMANN ❑YETTER

YOUR TA: ____

INSTRUCTIONS:
1. You have two hours for this examination.
2. You are permitted the use of a one page notes sheet (8.5x11, both sides).
3. Solve each problem in the space provided. Write the letter of your answer in the appropriate space on this page.
4. Show your work. A correct answer with no supporting work may receive little or no credit
5. Each problem is worth 10 points.
6. There are 18 problems; *you are to do all of them*.

Write the letters corresponding to your answers here:

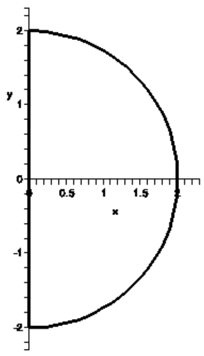
1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	
9.	
10.	
11.	
12.	
13.	
14.	
15.	
16.	
17.	
18.	

Score:__________________

Mathematics 114 Final Examination   Name______________________________
Spring, 2006   ID #______________________________

1. If R is the region inside the right half of the circle $x^2 + y^2 = 4$ (see diagram), then $\iint_R x\sqrt{x^2 + y^2}\ dA =$

a) 0   b) 2   c) 4   d) 8   e) $\frac{16}{3}$ f) 16



2. Evaluate: $\int_0^1 \int_0^2 (x + y)\ dxdy$

a) 3   b) 2   c) 3/2   d) 1   e) 1/2   f) 0

3. The set of points that are equidistant from the points (2, 6, 0) and (-4, 2, 4) is a plane. Find the equation of this plane.

a) $6x - 2y + 7z = 0$
b) $3x + 2y - 2z = 1$
c) $2x - 8y + 4z = 10$
d) $8x + 12y = 1$
e) $4x - 2y - 2z = 12$
f) $3x - y + z = 0$

4. Which of the following is true about the function $f(\text{x, y}) = 8x^3 + y^3 + 6xy$?

a) It has a local maximum, a local minimum, and no other critical points.
b) It has a saddle point, a local maximum, and no other critical points.
c) It has a saddle point, a local minimum and no other critical points.
d) It has two local minima and no other critical points.
e) It has two saddle points and no other critical points.
f) It has two local maxima and no other critical points.

5. Let S be the surface $x^2y + 4xz^3 - yz = 0$. An equation for the tangent plane to S at $(1, 2, -1)$ is
a) $y + 5z = -3$ b) $x - y + z = 0$ c) $2x + y + 5z = 0$
d) $x - 3z = 4$ e) $2x - 3y + z = 3$

6. Given $f(x, y) = x^2y^3$, $\mathbf{u} = < 3/5, -4/5 >$, the directional derivative $D_{\mathbf{u}}f$ in the direction of $\mathbf{u}$ is
a) $2xy^3 + 3x^2y^2$ b) $\dfrac{3x^2 - 4y^2}{5}$ c) $\dfrac{6x^2 - 12y^2}{5}$
d) $\dfrac{6xy^3 - 12x^2y^2}{5}$ e) $\sqrt{4x^2y^6 + 9x^4y^4}$

Mathematics 114 Final Examination Name____
Spring, 2006 ID #____

7. Calculate the length of the curve given parametrically by $x = t\cos t$, $y = t\sin t$, $z = \dfrac{2\sqrt{2}}{3}t^{3/2}$ for $0 \le t \le 2$. [Work carefully…the integrand really does simplify!]

a) $2\sqrt{2}$ b) $\sqrt{2}$ c) $6\sqrt{2}$ d) 4 e) 2 f) $\dfrac{8\sqrt{2}}{3}$

8. Let $f(x, y) = (x^3 + x)y^2$. Find the value of $f_{xy}(0, 1)$.

a) 0 b) 1 c) 2 d) 3 e) 5

Mathematics 114 Final Examination Name____
Spring, 2006 ID #____

9. $\lim\limits_{(x,y)\to(0,0)} \dfrac{(x+2y)}{\sqrt{x^2+4y^2}} = ?$

a) Does Not Exist b) 0 c) 1 d) $\dfrac{1}{\sqrt{2}}$ e) $\dfrac{3}{\sqrt{5}}$ f) ∞

10. Suppose that f is a differentiable function of x and y and that $\dfrac{\partial f}{\partial x} = e^{xy}$. Which of the following could be $\dfrac{\partial f}{\partial y}$?

a) e^{xy} b) $\dfrac{e^{xy}}{x}$ c) $\dfrac{e^{xy}}{y}$ d) $\dfrac{e^{xy}(xy-1)}{x^2}$ e) $\dfrac{e^{xy}(xy-1)}{y^2}$ f) $\dfrac{e^{xy}(x+y)}{x^2}$

11. A thermometer reading 70° is taken outside on a winter day when it is 34 degrees outside. Two minutes later, the thermometer reads 46°. According to Newton's Law of Cooling, what will be the reading on the thermometer two minutes after that?

a) 40° b) 39° c) 38° d) 37° e) 36° f) 35°

12. Reverse the order of integration to calculate $\int_0^8 \int_{\sqrt[3]{x}}^2 \frac{1}{y^4+1}\, dy\, dx$.

a) $\frac{1}{4}\ln 17$ b) $\frac{1}{2}\tan^{-1} 17$ c) $\frac{1}{4\sqrt{17}}$

d) $\frac{1}{4}\ln 65$ e) $\frac{1}{2}\tan^{-1} 65$ f) $\frac{1}{4\sqrt{65}}$

13. Suppose $y'' + 2y' + y = 0$, subject to $y(0) = -3$, $y'(0) = 0$. Which of the following is true about $y(x)$?

a) $y(1) = 0$ and $\lim_{x\to\infty} y(x) = 0$
b) $y(-1) = 0$ and $\lim_{x\to\infty} y(x) = 0$
c) $y(1) = 0$ and $\lim_{x\to-\infty} y(x) = 0$
d) $y(-1) = 0$ and $\lim_{x\to-\infty} y(x) = 0$
e) $y'(1) = 0$ and $\lim_{x\to\infty} y(x) = 0$
f) $y'(-1) = 0$ and $\lim_{x\to\infty} y(x) = 0$

14. What is the coefficient of x^5 in the series solution of the initial-value problem $y'' + xy' + y = 0$, $y(0) = 0$, $y'(0) = 1$?

(a) 1/3 (b) -1/3 (c) 1/15 (d) -1/15 (e) 1/60 (f) -1/60

15. Solve $y' + 3x^2 y = 6x^2$ subject to $y(0)=3$.

a) $y = x^2+3$ b) $y = x^3 - x^2 + 3$ c) $y = \dfrac{x+6}{3x+2}$

d) $y = \dfrac{x^2-3}{x^2-1}$ e) $y = 1 + 2e^{x^2}$ f) $y = 2 + e^{-x^3}$

16. Let $H = \left\{(x,y,z) \mid x^2 + y^2 + z^2 \leq 1,\ z \geq 0\right\}$. Then $\iiint_H z^2 dV = ?$

a) $2\pi/15$ b) $2\pi/5$ c) $2\pi/3$ d) $\pi/3$ e) $\pi/2$ f) π

17. Find a particular solution for $y'' + y' = \sin x$

a) $\sin x$ b) $\cos x$ c) $\sin x + \cos x$

d) $\dfrac{\sin x + \cos x}{2}$ e) $-\dfrac{\sin x + \cos x}{2}$ e) $\dfrac{-\sin x + \cos x}{2}$

18. A solid body occupies the region bounded by the xy-plane, the plane $z = 1$ and the cylinder $x^2 + y^2 = 1$. Its density varies linearly from top to bottom, being given by $\rho(x,y,z) = z + 4$. The mass of the body is:

a) 9π b) $9\pi/2$ c) $9\pi/4$ d) π e) 9 f) 9/2 g) 9/4