Math 114: Final Exam

May 4, 2012

Instructions:

1. Please sign your name and indicate the name of your instructor and your teaching assistant:

   A. Your Name:

   B. Your Instructor:

   C. Your Teaching Assistant:

2. This exam is 2 hours long and there are 16 questions.
3. You can use one handwritten two-sided page of notes; no books or calculators are allowed.
4. **It is important that you show your work for each problem.** To receive credit for a problem, you must both indicate the correct answer and show plausible work justifying your answer.
5. Do not come to the front of the class when the exam is over; we will pick up your exam from you.
6. **Don't take any work with you which is needed to justify your answers.**
7. If you want to find out how you did on the exam, record your answers on the last page of the exam, tear this off and take it with you. We will post an answer key on the web.

The table below is for grading purposes only. You do not need to transfer your answers to this page.

1.	2.	3.	4.	5.	
6.	7.	8.	9.	10.	
11.	12.	13.	14.	15.	16.

Total:

(1) Suppose that the function $y = y(t)$ is a solution of the autonomous differential equation

$$\frac{dy}{dt} = -y \cdot (y-1) \cdot (y-2)$$

What are **all** values of y which are **stable** equilibrium values?

(A) $y = 1$ only

(B) $y = 0$ only

(C) $y = 2$ only

(D) $y = 0$ and $y = 2$

(E) $y = 0$ and $y = 1$ and $y = 2$

(F) none of the above

Answer to 1:

(2) Suppose C is a smooth simple closed curve in the plane which bounds a region R. Which of the following path integrals must always equal **twice** the area of R? (The path integral is taken counterclockwise around C.)

(A) $\oint xdy + ydx$

(B) $\oint xdy - ydx$

(C) $\oint ydy - xdx$

(D) $\oint ydy + xdx$

(E) $2 \oint ydy - xdx$

(F) None of the above.

Answer to 2:

(3) Which of the following vector fields $\mathbf{F} : \mathbb{R}^2 \to \mathbb{R}^2$ in the plane are conservative, in the sense that the value of a path integral of the vector field always just depends on the endpoints of the path? The formulas below express the value of $\mathbf{F}$ at a given point (x, y) as a linear combination of $\mathbf{i} = (1, 0)$ and $\mathbf{j} = (0, 1)$

(I) $\mathbf{F} = x\ \mathbf{i} + y\ \mathbf{j}$.

(II) $\mathbf{F} = y\ \mathbf{i} + x\ \mathbf{j}$.

(III) $\mathbf{F} = y^2\ \mathbf{i} + x^2\ \mathbf{j}$.

(A) I only
(B) II only
(C) III only
(D) I and III
(E) I and II
(F) none of the above

Answer to 3:

(4) Find the area of the parallelogram in three space whose vertices are at $(0, 0, 0)$, $(1, 1, 1)$, $(1, -1, 0)$, $(2, 0, 1)$.

(A) $\sqrt{6}$

(B) $\sqrt{3}$

(C) 1

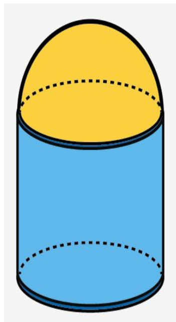
(D) 2

(E) 4

(F) none of the above

Answer to 4:

(5) A silo is to be built having a flat circular bottom, cylindrical side, and a hemispherical top.



The silo is to have total volume (including the top cap) $V = 900\pi$ cubic feet. If the cost of the metal is 5 dollars/sq.ft. for the roof, 2 dollars/sq.ft. for the sides, and 1 dollar/sq.ft. for the floor, what should the ratio of height (h) to radius (r) of the cylindrical portion of the silo be in order to minimize the total cost of the materials needed?

You can use without proof the usual formulas for the volume and surface areas of spheres and cylinders. For example, a sphere of radius r has volume $4\pi r^3/3$ and surface area $4\pi r^2$. The volume of a solid cylinder of radius r and height h is $\pi r^2 h$. The surface area of the flat bottom of such a cylinder is πr^2, while the surface area of the curved vertical part of the cylinder is equal to $2\pi rh$.

(A) $h : r = 1 : 1$

(B) $h : r = 3 : 2$

(C) $h : r = 4 : 3$

(D) $h : r = 5 : 3$

(E) $h : r = 5 : 2$

(F) $h : r = 7 : 2$

(G) $h : r = 9 : 5$

(H) None of the above

Answer to 5:

(6) Consider the spiraling curve in parametric cylindrical coordinates $r(t) = t$, $\theta(t) = \pi t$, $z(t) = t^2$. Which of the following computes the length of this curve from the origin to $(2, 0, 4)$?

(A) $\displaystyle\int_{t=0}^{2\pi} \sqrt{1 + (\pi^2 + 4)t^2}\, dt$

(B) $\displaystyle\int_{t=0}^{2} \sqrt{1 + (\pi^2 + 4)t^2}\, dt$

(C) $\displaystyle\int_{t=0}^{2\pi} \sqrt{1 + \pi^2 + 4t^2}\, dt$

(D) $\displaystyle\int_{t=0}^{2} \sqrt{1 + \pi^2 + 4t^2}\, dt$

(E) $\displaystyle\int_{t=0}^{4} t\sqrt{1 + \pi^2 + t^2}\, dt$

(F) $\displaystyle\int_{t=0}^{2} t\sqrt{1 + \pi^2 + t^2}\, dt$

(G) $\displaystyle\int_{t=0}^{2\pi} t\sqrt{1 + \pi^2 + t^2}\, dt$

(H) None of the above.

Answer to 6:

(7) A spaceship flies along the parameterized curve $0 \leq t < \infty$

$$x(t) = e^t \quad ; \quad y(t) = (t-1)^2 \quad ; \quad z(t) = 1 + t$$

The temperature of space is given by $\tau(x, y, z) = xyz$. At the exact point at which the spaceship is **coldest** along its path, what is the dot product of the spaceship's velocity vector $\hat{v}$ with the gradient vector $\nabla\tau$ of temperature?

(A) 0

(B) $\frac{1}{2}$

(C) $-\frac{1}{2}$

(D) 1

(E) $\frac{1}{e}$

(F) e

(G) $e\sqrt{2}$

(H) None of the above.

Answer to 7:

(8) Which of the following is the equation of the tangent plane to the surface

$$x^3 - xy^2 + 4xz = 9$$

when $x = 1$ and $y = 2$?

(A) $11x - 4y + 4z = 9$

(B) $-x - 4y + 4z = 0$

(C) $11x - 4y + 4z = 0$

(D) $x + 2y + 3z = 9$

(E) $x + 2y + 3z = 0$

(F) $x + 2y + 3z = 23$

(G) $11x - 4y + 4z = 15$

(H) None of the above.

Answer to 8:

(9) A particle in space accelerates according to $\vec{a}(t) = 2\mathbf{i} + (t^2 - 1)\mathbf{j} + \mathbf{k}$ with initial velocity $\vec{v}_0 = 3\mathbf{i} + 4\mathbf{j}$ and initial position $\vec{r}_0(t) = \mathbf{i} + 5\mathbf{k}$. Which of the following is its position at time $t = 2$? (Recall that $\mathbf{i} = (1, 0, 0)$, $\mathbf{j} = (0, 1, 0)$ and $\mathbf{k} = (0, 0, 1)$.)

(A) $4\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$

(B) $2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$

(C) $11\mathbf{i} + 7\frac{1}{3}\mathbf{j} + 7\mathbf{k}$

(D) $3\mathbf{i} + 9\mathbf{j} + 6\mathbf{k}$

(E) $\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}$

(F) $\mathbf{i} - \frac{2}{3}\mathbf{j} + \mathbf{k}$

(G) $2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$

(H) None of the above.

Answer to 9:

(10) Evaluate $\iint_R e^{-y^2}\, dA$, where R is the triangular region with vertices $(0,0)$, $(0,1)$ and $(1,1)$.

(A) $\frac{3}{4}(1-e^{-2})$ (B) $\frac{1}{3}(1-e^{-2})$ (C) $\frac{1}{2}(1-e^{-2})$

(D) $(1-e^{-2})$ (E) $\frac{3}{2}(1-e^{-1})$ (F) $\frac{1}{2}(1-e^{-1})$

(G) $\frac{2}{3}(1-e^{-1})$ (H) $2(1-e^{-1})$

Answer to 10

(11) Evaluate $\iiint_D e^{x-y+z}\, dV$, where D is the parallelepiped bounded by the planes:
$x - y + z = 2, \quad x - y + z = 3, \quad x + 2y = -2, \quad x + 2y = 1, \quad x - z = 0, \quad x - z = 2.$

(A) $\frac{4}{5}(1 - e^2)$ (B) $\frac{1}{5}(e^3 - e^2)$ (C) $\frac{4}{5}(e^3 - e^4)$

(D) $\frac{1}{5}(e^4 - 1)$ (E) $\frac{3}{5}(1 - e^2)$ (F) $\frac{6}{5}(e^3 - e^2)$

(G) $\frac{3}{5}(e^3 - e^4)$ (H) $\frac{6}{5}(e^4 - 1)$

Answer to 11:

(12) A solid cylinder of height 1 and radius 1 is formed by all the points (x, y, z) such that $x^2+y^2 \leq 1$ and $0 \leq z \leq 1$. Find the center of mass of the cylinder if its density at (x, y, z) is given by $\rho(x, y, z) = z$.

(A) $(0, 0, 1)$ (B) $(0, 0, 1/2)$ (C) $(0, 0, 2/3)$

(D) $(0, 0, 3/4)$ (E) $(0, 0, \pi/2)$ (F) $(0, 0, \pi/3)$

(G) $(0, 0, 2\pi/3)$ (H) $(0, 0, 3\pi/4)$

Answer to 12:

(13) Calculate the work done by the force field $F = \langle y^2 \, , \, x \rangle$ on a particle which moves along the parabola $y = x^2$ from the point $(0, 0)$ to the point $(1, 1)$.

(A) 2/5	(B) 1/3	(C) 7/15	(D) 3/5
(E) 2/3	(F) 11/15	(G) 4/5	(H) 13/15

Answer to 13:

(14) Suppose $f(x,y,z) = xe^{2yz}$. A unit length vector $\mathbf{V}$ in the direction of fastest *decrease* of f at the point (1,1,0) is:

(A) $\mathbf{V} = -\mathbf{i}$

(B) $\mathbf{V} = \mathbf{i} + 2\mathbf{k}$

(C) $\mathbf{V} = -\mathbf{i} - \mathbf{k}$

(D) $\mathbf{V} = \dfrac{1}{\sqrt{5}}(\mathbf{i} + 2\mathbf{k})$

(E) $\mathbf{V} = -\dfrac{1}{\sqrt{5}}(\mathbf{i} + 2\mathbf{k})$

(F) $\mathbf{V} = \dfrac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{k})$

(G) $\mathbf{V} = -\dfrac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{k})$

(H) $\mathbf{V} = \dfrac{1}{\sqrt{2}}(\mathbf{i} - \mathbf{k})$

Answer to 14:

(15) The function $f(x, y) = xy^2 + x^2y + 12x$ has:

(A) No critical points at all.
(B) Exactly one saddle point and no other critical points.
(C) Exactly one local maximum and no other critical points.
(D) Exactly one local minimum and no other critical points.
(E) One local maximum and one saddle point.
(F) One local maximum and one local minimum.
(G) One local minimum and one saddle point.
(H) Two saddle points.

Answer to 15:

(16) If three resistors having resistances x, y and z are connected in parallel, then the resistance R of the entire circuit satisfies the equation

$$\frac{1}{R} = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}.$$

Use differentials to estimate R to three decimal places if $x = 2.012$, $y = 4.008$ and $z = 3.992$. (Note, $1 = \frac{1}{2} + \frac{1}{4} + \frac{1}{4}$.)

(A) 0.998	(B) 0.999	(C) 1.000	(D) 1.001
(E) 1.002	(F) 1.003	(G) 1.004	(H) 1.005

Answer to 16:

Math 114: Final Exam Answer Record

Answer Record Instructions:

(1) **Record your answers on this sheet.**

(2) **Tear off this sheet and take it with you after the exam.**

(3) **Check your answers against the key to be posted on your courses web page.**

Answers

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