MATH 114 **Final Exam** **Fall- 2012**

Problem 1. Assume the acceleration of gravity is $10\,^{\text{metrs}}/_{\text{sec}^2}$ downwards. A ball is hit with a horizontal velocity of $20\,^{\text{meters}}/_{\text{sec}}$ and a vertical upward velocity of $25\,^{\text{meters}}/_{\text{sec}}$. If the ball is initially 1 meterabove ground when it is hit, how high a fence will the ball clear if the fence is 80 metersaway from where the ball is hit?

Answer (in meters):
a) 1 b) 6 c) 11 d) 16 [e) 21] f) 26 g) 31 h) 36

Problem 2. If

$$\frac{dy}{dt} = k(3 - y)$$

where k is a constant and $y(0) = 0$ and $y(1) = 1$, what is $y(2)$?

Answer:
a) -1 b) $-\frac{1}{3}$ c) 0 d) $\frac{1}{2}$ e) $\frac{3}{5}$ f) 1 [g) $\frac{5}{3}$] h) $\frac{10}{3}$

Problem 3. Find the $z-$coordinate of the point on the surface $2z = 10 - 4x^2 - 6y^2$ where the tangent plane is parallel to the plane $4x + 12y + 2z = 0$.

Answer:
a) -2 b) -1 c) 0 d) $\frac{1}{2}$ e) 1 [f) $\frac{3}{2}$] g) 2 h) $\frac{5}{2}$

Problem 4. The velocity v of a particle moving along the $x-$axis (so $v = \frac{dx}{dt}$) satisfies the differential equation

$$\frac{dv}{dt} = -\frac{v}{4}.$$

What is the position of the particle at time $t = 4$ seconds, if you know that the initial postion $x(0) = 0$, and the initial velocity $v(0) = \frac{dx}{dt}(0) = 5$ feet/sec.

Answer (in feet):
a) $\frac{10}{e}$ b) $10 - \frac{5}{e}$ c) $12 + \frac{2}{e}$ [d) $20 - \frac{20}{e}$] e) 24 f) $15 - \frac{15}{e}$ g) 40 h) $20 + \frac{2}{e}$

Problem 5. Find the distance from the ellipsoid $x^2 + y^2 + 4z^2 = 4$ to the plane $x + y + z = 6$.

(Hint: Write down the distance from a point to the plane, and minimize it as the point varies on the ellipsoid.)

Answer:
a) 0 b) $\frac{1}{\sqrt{6}}$ c) $\frac{1}{\sqrt{2}}$ d) $\frac{2}{3}$ e) 1 [f) $\sqrt{3}$] g) 2 h) $\sqrt{6}$

Problem 6. Compute the double integral

$$\iint_R \frac{3x-y}{3x+y}\,dA,$$

where R is the region inside the triangle with vertices $(0,0)$, $(1,3)$ and $(2,0)$.

Answer:

$\boxed{\text{a) } \frac{3}{2}}$ b) $\frac{5}{4}$ c) 1 d) $\frac{5}{4}$ e) $\frac{2}{3}$ f) $\frac{1}{2}$ g) $\frac{1}{3}$ h) 0

Problem 7. Compute the double integral

$$\int_0^4 \int_{1-\frac{y}{4}}^1 \cos(x^2)\,dx\,dy.$$

Answer:

a) 3π b) $3\cos(1)$ c) $\sin(2)$ $\boxed{\text{d) } 2\sin(1)}$ e) $\pi - 1$ f) $2-\sin(1)$ g) 4 h) 0

Problem 8. Find the sum of the maximum and minimum of the curvature of the ellipse

$$9(x-1)^2 + y^2 = 9.$$

(*Hint.* The ellipse can be parametrised $(x(t), y(t))$ with $x(t) = 1+\cos(t)$. Find $y(t)$ and sketch the ellipse to find where the curvature is greatest and least.)

Answer:

a) 0 b) 1 c) 3 d) 9 e) $\frac{10}{9}$ f) $\frac{4}{3}$ $\boxed{\text{g) } \frac{28}{9}}$ h) $\frac{28}{3}$

Problem 9. Compute the triple integral

$$\iiint_R z\,dV,$$

where R is the region inside the sphere of radius 2 centered at the origin in the first octant. (So $R = \{\,(x,y,z)\mid x^2+y^2+z^2 \le 4,\ x \ge 0,\ y \ge 0,\ z \ge 0\,\}$.

Answer:

$\boxed{\text{a) } \pi}$ b) $2\pi(1-\frac{1}{\sqrt{2}})$ c) 3 d) $2\sqrt{2}$ e) 3π f) 7π g) $\frac{11\pi}{4}$ h) 10π

Problem 10. What is $\oint_C x dy - y dx$, where C is the curve composed of a straight line segment from $(-2,0)$ to $(0,0)$, a straight line segment from $(0,0)$ to $(0,-2)$, and the part of the circle of radius 2, centered at the origin, traversed counterclockwise starting from $(0,-2)$ and ending at $(-2,0)$

Answer:

a) -1 b) $-\pi$ c) 0 d) 2π e) 3π $\boxed{\text{f) } 6\pi}$ g) $\frac{9\pi}{4}$ h) 9π

Problem 11. Let f be the function $f(x, y) = x^2 - 2x + 2y^2$. Find the sum of the maximum and the minimum of f inside the region $x^2 + y^2 \leq 4$.

Answer:

a) -2 b) -1 c) 0 d) 1 e) 4 f) 8 g) 12 h) 20

Problem 12. What is the limit

$$\lim_{(x,y)\to(0,0)} \frac{(x^2+y^2)\sin(x^2+y^2)}{x^4+y^4}.$$

Answer:

a) 2 b) 1 c) 3 d) 0 e) Does not exist f) 6 g) $\frac{9}{4}$ h) 9

Problem 13. The maximal volume of a parallelipiped whose sides a, b, c satisfy $ab+4ac+9bc \leq 432$ is:

Answer:

a) 260 b) 264 c) 268 d) 272 e) 276 f) 280 g) 284 h) 288

Problem 14. The function $h(x, y) = 2x\sin(y) + y^2 - x^2$ has exactly:

Answer:

a) one local maximum, one local minimum, and no saddle points
b) two local maxima
c) two local minima
d) one saddle point and one local minimum
e) one saddle point
f) one saddle point and one local maximum
g) one local minimum
h) one local maximum

Problem 15.

i) For what values of $n = 1, 2, \ldots,$ is the vector field

$$\vec{F} = \frac{x}{(x^2+y^2)^n}\vec{i} + \frac{y}{(x^2+y^2)^n}\vec{j}$$

conservative?

ii) For what values of $n = 1, 2, \ldots,$ is the vector field

$$\vec{F} = \frac{x}{(x^2+y^2)^n}\vec{i} - \frac{y}{(x^2+y^2)^n}\vec{j}$$

conservative?

Proof. i) For all values of n: If $n \neq 1$, then $\vec{F} = \nabla \left(\frac{1}{2(1-n)} (x^2+y^2)^{1-n} \right)$, and similarly for $n = 1$, with potential function $\frac{1}{2} \ln(x^2+y^2)$

Note that since the domain is not simply connected it is not enough to check closedness.

ii) For no values of n: If $\vec{F}$ where conservative for some n, then because of i) so would be the sum, that is $\frac{2x}{(x^2+y^2)^n} \vec{\imath}$ would be conservative, which is obviously false. □

Problem 16. TRUE or FALSE. For each of the following statements, indicate whether it is true (T) or false (F). Support your answers.

i The function $g(x,y) = \begin{cases} \frac{x^3+y^3}{x+y} & \text{if } x \neq -y \\ 0 & \text{if } x = -y \end{cases}$ is continuous.

ii The curvature of the curve $\langle 4t, \cos(2t), \sin(2t) \rangle$ is constant.

iii Let $\vec{F}(x,y) = -2e^{2x+y}\vec{\imath} + e^{2x+y}\vec{\jmath}$. The line integral $\oint_C \vec{F} \cdot d\vec{r}$ along any simple closed positively (counterclockwise) oriented loop is negative.

iv There is a solution of the differential equation $\frac{dy}{dt} = \sin(y)$ whose graph is a line.

Answer:

i FALSE

ii TRUE

iii FALSE

iv TRUE