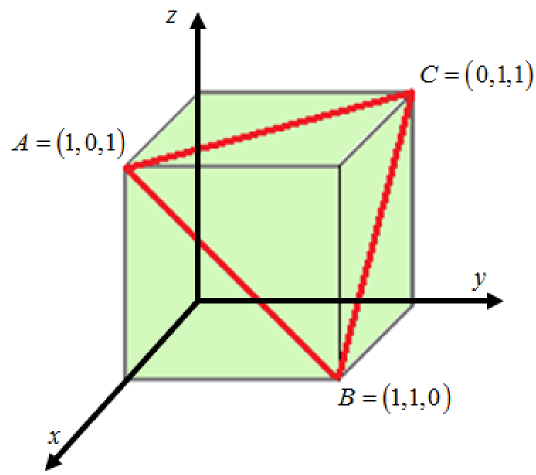
University of Pennsylvania **Math 114 Final Exam Fall 2010**
Department of Mathematics **Haglund, Gupta, Powers, and Rimmer**

1. Find the components of the vector from the point A to the midpoint of line segment $\overline{BC}$.

A. $\left\langle 1, \frac{1}{2}, \frac{1}{2} \right\rangle$ B. $\left\langle 1, \frac{-1}{2}, \frac{-1}{2} \right\rangle$ C. $\left\langle \frac{1}{2}, 1, \frac{1}{2} \right\rangle$ D. $\langle 1, 0, 1 \rangle$

E. $\left\langle 1, \frac{1}{2}, 1 \right\rangle$ F. $\left\langle 0, \frac{1}{2}, \frac{-1}{2} \right\rangle$ G. $\left\langle \frac{-1}{2}, 1, \frac{-1}{2} \right\rangle$ H. $\left\langle 1, \frac{-1}{2}, -1 \right\rangle$



2. If y satisfies the differential equation $\frac{dy}{dt} = -k(y+1)$ where k is a positive constant. Given that $y(0) = 2$ and $y(1) = 1$ what is $y(2)$? Answer $y(2) =$

A. $\frac{-1}{2}$ B. $\frac{-1}{3}$ C. 0 D. $\frac{1}{3}$ E. $\frac{1}{2}$ F. $\frac{2}{3}$ G. $\frac{3}{4}$ H. 1

3. Find the value of the x-coordinate where the plane through the points $(x, y, z) = (4,1,1), (1,2,1)$, and $(1,1,2)$ intersects the $x-$axis. Answer $x =$

A. 14 B. 10 C. 8 D. 5 E. 3 F. 1 G. 0 H. -2

4. Find the x-coordinate of the point on the plane $x - 2y + z = 3$ that is closest to the point $(x, y, z) = (1,1,1)$. Answer $x =$

A. -1 B. $\frac{-1}{2}$ C. 0 D. $\frac{1}{2}$ E. 1 F. $\frac{3}{2}$ G. 2 H. $\frac{5}{2}$

5. The function $z = f(x, y)$ is given implicitly by the equation $z^3 + z = x^2 + y^2$. Note that when $x = 1$ and $y = 1$, $z = 1$ as well. Compute $\frac{\partial f}{\partial x}(1,1)$.

A. $\frac{-3}{2}$ B. -1 C. $\frac{-1}{2}$ D. 0 E. $\frac{1}{2}$ F. 1 G. $\frac{3}{2}$ H. 2

6. Consider the surface $z = x^2 + x + 2y^2$. At what point (x_0, y_0, z_0) is the tangent plane parallel to the plane $x + 4y + z = 0$. What is the z coordinate of that point?
Answer $z_0 =$
A. -1 B. 0 C. 1 D. 2 E. 3 F. 4 G. 6 H. 7

7. Let $f(x, y, z) = zx - xy^2$. At the point $(1,1,1)$, find the angle between the vector pointing in the direction of fastest increase of $f(x, y, z)$ and the $x-$axis.

A. -1 B. $\frac{-1}{2}$ C. 0 D. 0 E. $\frac{\pi}{6}$ F. $\frac{\pi}{4}$ G. $\frac{\pi}{3}$ H. $\frac{\pi}{2}$

8. A cannon placed on a wall 64 feet above the ground fires a cannon ball level in the horizontal direction with horizontal velocity $80 \frac{\text{ft.}}{\text{s.}}$. How far (the horizontal distance) from the foot of the wall does the cannon ball land when it hits the ground? Assume the acceleration due to gravity is $32 \frac{\text{ft.}}{\text{s.}^2}$. Distance =
A. 32 ft B. 48 ft C. 64 ft D. 80 ft E. 120 ft F. 150 ft G. 160 ft H. 200 ft

9. Let $\mathbf{r}(t) = \langle 2t, t^2, \ln t \rangle$. Find the arclength for $1 \le t \le e$. Arclength =

A. 1 B. $\ln 2$ C. 2 D. $e-1$ E. e F. e^2 G. 12 H. 16

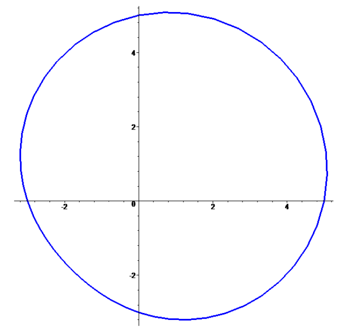
10. Find the maximum curvature of the curve $\mathbf{r}(t) = \langle t, t, t^2 \rangle$.
A. 1 B. $\frac{1}{\sqrt{2}}$ C. $\frac{1}{\sqrt{3}}$ D. $\frac{1}{2}$ E. $\frac{1}{2\sqrt{2}}$ F. $\frac{4}{7}$ G. $\frac{1}{\sqrt{13}}$ H. 0

11. Find the minimum of $f(x, y, z) = xy + 2xz + 3yz$ subject to the constraint $xyz = 6$. With $x \ge 0, y \ge 0,$ and $z \ge 0,$ the minimum is
A. 24 B. 18 C. 13 D. 12 E. 10 F. 8 G. 6 H. 3

12. Evaluate $I = \int_0^4 \int_{\sqrt{x}}^2 \sin\left(y^3\right) dy dx$. $I =$

A. -1 B. 0 C. 1 D. $\cos 4$ E. $2\cos 8$ F. $4\cos 2$ G. $\frac{1}{2}\cos 4$ H. $\frac{1}{3}(1-\cos 8)$

13. Find the area enclosed by the curve given in polar coordinates by $r(\theta) = 4 + \sin\theta + \cos\theta$ with $0 \le \theta \le 2\pi$.



A. 64π B. 17π C. $\sqrt{17}\pi$ D. 16π E. $30\pi\left(\sqrt{2}-1\right)$

F. 11π G. $7\sqrt{3}\pi$ H. $\frac{16\pi}{3}$

14. Find the volume of the region R inside the sphere of radius 2 and above the cone $\sqrt{3}z = \sqrt{x^2+y^2}$ i.e, $\left(x^2+y^2+z^2 \le 4 \text{ and } \sqrt{3}z \ge \sqrt{x^2+y^2}\right)$. Volume =

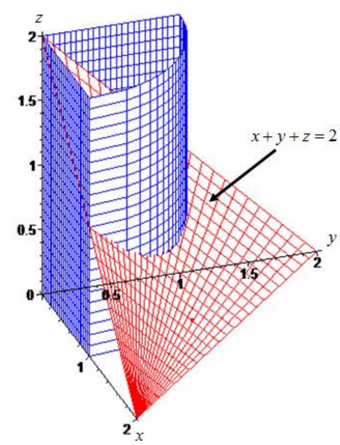
A. $\frac{8\pi}{3}$ B. $\frac{4\pi\left(\sqrt{2}-1\right)}{3}$ C. $\frac{8\pi\left(2-\sqrt{3}\right)}{3}$ D. $\frac{7\pi}{2}$ E. $4\sqrt{2}\,\pi$ F. $6\pi\left(\sqrt{3}-1\right)$

G. 4π H. $\left(\sqrt{2}-1\right)\pi$

15. Find the volume inside the cylinder $x^2+y^2=1$, below the plane $x+y+z=2$, above the $xy-$plane, and in the first octant $\left(x^2+y^2 \le 1, x+y+z \le 2, x \ge 0, y \ge 0, \text{ and } z \ge 0\right)$. Volume =

A. $\frac{\pi-1}{6}$ B. $\frac{\pi}{3}$ C. $\frac{\pi}{6}-\frac{1}{12}$ D. $\frac{\pi}{6}$

E. $\frac{\pi}{4}-\frac{1}{6}$ F. $\frac{\pi}{6}+\frac{1}{2}$ G. $\frac{\pi}{2}-\frac{2}{3}$ H. π



16. Evaluate $\iint_S (x+y)e^{x^2-y^2}\,dA$ where S is the rectangle with vertices $(1,0),(0,1),\left(-\frac{1}{2},\frac{1}{2}\right)$, and $\left(\frac{1}{2},-\frac{1}{2}\right)$. Note: $x^2-y^2=(x+y)(x-y)$.

A. $2e$ B. e C. $\frac{e}{2}$ D. $\frac{1}{e}$ E. $e-\frac{1}{e}$ F. $\frac{1}{2e}$ G. $\frac{1}{2}\left(e+\frac{1}{e}-2\right)$ H. 0

17. Evaluate $\int_C x^2dx+y^2dy+z^2dz$ where C is the straight line segment from $(1,2,3)$ to $(2,3,4)$.

A. 30 B. 24 C. 21 D. 20 E. 16 F. 14 G. 8 H. 0

18. Find the value of the line integral $I=\int_C \left(x^2+y\right)dx+\left(y^2-x\right)dy$ where C is the triangle with vertices $(x,y)=(0,0),(3,0),(0,4)$ traversed counterclockwise. $I=$

A. 10 B. 7 C. 6 D. 0 E. -3 F. -5 G. -8 H. -12

Answers:

1. G	**10. A**
2. D	**11. B**
3. B	**12. H**
4. F	**13. B**
5. E	**14. A**
6. D	**15. G**
7. H	**16. G**
8. G	**17. C**
9. F	**18. H**