Math 114, FINAL EXAM

December 21, 2009

INSTRUCTIONS

INSTRUCTIONS:

1. Please complete the information requested below. There are 9 multiple choice problems and 4 open answer problems. No partial credit will be given on the multiple choice questions.
2. Please show all your work on the exam itself. Correct answers with little or no supporting work will not be given credit.
4. You are allowed to use one hand-written sheet of paper with formulas. *No calculators, books or other aids are allowed.* Please turn in your crib sheet together with your exam.

• **Name** (please print):

• **Name of your professor:**
◯ **Dr. Haglund** ◯ **Dr. Pantev**
◯ **Dr. Pimsner** ◯ **Dr. Zhu**

• **Name of your TA:**
◯ **Victoria Behrend** ◯ **John Cui** ◯ **Eric Korman**
◯ **Shanshan Ding** ◯ **Alberto Garcia-Raboso** ◯ **Alex Liu**
◯ **Victor Lu**

• **I certify that all of the work on this test is my own.**
Signature:

• **Recitation day and time:**

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Problem	Points
1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	
9.	
10.	
11.	
12.	
13.	
Total	

Part I: Multiple choice questions

1. [10 points] Find the angle between the xy-plane and the tangent plane to the ellipsoid

$$\frac{x^2}{12} + \frac{y^2}{12} + \frac{z^2}{3} = 1$$

at the point $(2, 2, 1)$.

(A) $\frac{\pi}{3}$ (B) $\arccos\left(\sqrt{\frac{2}{3}}\right)$ (C) $\frac{\pi}{2}$

(D) $\arccos\left(\sqrt{\frac{11}{12}}\right)$ (E) $\frac{\pi}{6}$ (F) $\arccos\left(\sqrt{\frac{1}{27}}\right)$

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2. [10 points] Find the distance from the origin to the plane passing through the point $(1, 0, 1)$ and containing the line $\langle t, 1, 5 - t\rangle$.

(A) 2 (B) $\frac{2}{\sqrt{11}}$ (C) $\sqrt{2}$

(D) $2\sqrt{2}$ (E) $\frac{2}{3}$ (F) none of the above

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3. 10 points A news helicopter is descending along the helix $\langle \sin(\pi t), \cos(\pi t), 10 - t \rangle$. At time $t = 5$ the crew turns on a powerful head light shining straight ahead in the direction of the velocity vector. What spot on the ground, i.e. what point on the xy-plane, does this beam of light hit?

(A) $(-\pi, 0)$ (B) $(0, -1)$ (C) $(0, 0)$

(D) $(-5\pi, -1)$ (E) $(\pi, 5)$ (F) none of the above

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4. 10 points Let M be the absolute maximum of $f(x, y, z) = xyz$ in the sphere $x^2 + y^2 + z^2 \leq 3$ and let m be the absolute minimum. What is $M + m$?

(A) 1 (B) -1 (C) 0

(D) 2 (E) -2 (F) none of the above

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5. 10 points Find the mass of the sphere of radius a if its density at any point is proportional to the distance to the center of the sphere with proportionality coefficient k.

(A) $4\pi ka$ (B) $\frac{ka^3}{27}$ (C) πka^4

(D) $2\pi ka^3$ (E) $1 + 2\pi ka^4$ (F) none of the above

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6. 10 points Suppose $z = f(x, y)$, where $x = g(s, t)$, $y = h(s, t)$. Suppose that we know that

$$g(1,2) = 3, \quad g_s(1,2) = -1, \quad g_t(1,2) = 4,$$
$$h(1,2) = 6, \quad h_s(1,2) = -5, \quad h_t(1,2) = 10,$$
$$f_x(3,6) = 7, \quad f_y(3,6) = 8.$$

Find $\frac{\partial z}{\partial s} + \frac{\partial z}{\partial t}$ when $s = 1$ and $t = 2$.

(A) 60 (B) -60 (C) 61

(D) -61 (E) 62 (F) -62

7. 10 points A kid is riding a roller coaster in an amusement park. Part of the track follows the curve

$$\overrightarrow{r}(t) = \left\langle t,\ t^2,\ \frac{2}{3}t^3 \right\rangle, \qquad 0 \le t \le 2$$

How long is this part of the coaster track?

(A) $\frac{22}{3}$ (B) $\frac{1}{\sqrt{3}}$ (C) 1

(D) $\frac{5}{3}$ (E) 8 (F) none of the above

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8. 10 points Evaluate

$$\int_0^1 \int_{\sqrt{y}}^1 \frac{ye^{x^2}}{x^3} dxdy$$

(A) $\frac{1}{4}(e-1)$ (B) $\frac{1}{2}\left(1-\frac{1}{e}\right)$ (C) -1

(D) $\frac{5}{3}$ (E) $-\frac{2}{e}$ (F) none of the above

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9. 10 points The force field

$$\overrightarrow{F} = \langle 2xy, x^2 + y \rangle$$

acts on an object moving along the path

$$\overrightarrow{r}(t) = \left\langle t^2 e^{(t-2)^3}, \frac{t}{\sqrt{t+2}} \right\rangle, \qquad 0 \le t \le 2.$$

Find the work done by $\overrightarrow{F}$.

(A) 1 (B) 2 (C) $\frac{1}{2}$

(D) $\frac{33}{2}$ (E) $\frac{2}{9}$ (F) none of the above

Part II: Open answer questions

10. [10 points] Solve the initial value problem

$$xy' - 2y = x^2, \qquad y(-1) = 0.$$

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11. [10 points] Let R be the region in the plane bounded by the square with vertices $(0,1)$, $(1,2)$, $(2,1)$, and $(1,0)$. Evaluate the integral

$$\int\int_R (x+y)^2 sin(x-y)dA.$$

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12. [10 points] True or false. Give a reason or a counterexample

(a) If $\overrightarrow{a}$ is a non-zero vector in three space, then $\text{proj}_{\overrightarrow{a}\times\overrightarrow{k}}(\overrightarrow{a}) = \overrightarrow{0}$.

(b) The vector $(\,\widehat{\mathbf{j}} \times (\,\widehat{\mathbf{k}} \times\, \widehat{\mathbf{j}})) \times\, \widehat{\mathbf{i}}$ is a unit vector.

(c) If $\overrightarrow{a}$ and $\overrightarrow{b}$ are perpendicular non-zero vectors, then $3\overrightarrow{a} + 2\overrightarrow{b}$ and $-3\overrightarrow{a} + 2\overrightarrow{b}$ have the same length.

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13. [10 points] Let C be the positively oriented boundary of the region D in the upper half plane that lies between the circle centered at the origin of radius 1 and the circle centered

at the origin of radius 3. Evaluate the integral

$$\int_C \left(\arctan\left(x^2\right) + y^2\right) dx + \left(e^{y^2} - x^2\right) dy.$$

Answer Key:

1. (B)
2. (B)
3. (D)
4. (C)
5. (C)
6. (C)
7. (A)
8. (A)
9. (D)
10. $x^2 \ln(|x|)$
11. 0
12. (a) is true, (b) is true, (c) is true
13. $-104/3$