Math 114: Final Exam

12 p.m., Dec. 17, 2008

Instructions:

1. Please print your name and indicate the name of your instructor and your teaching assistant:

   A. Your Name:

   B. Your Instructor:

   C. Your Teaching Assistant:

2. This exam is 2 hours long and there are 20 questions.
3. You can use one handwritten two-sided page of notes, or two handwritten one sided pages of notes. No books or calculators are allowed.
4. **It is important that you show your work for each problem.** To receive credit for a problem, you must both indicate the correct answer and show plausible work justifying your answer.
5. Do not come to the front of the class when the exam is over; we will pick up your exam from you.
6. **Don't take any work with you which is needed to justify your answers.**

The table below is for grading purposes only. You do not need to transfer your answers to this page.

1.	2.	3.	4.	5.
6.	7.	8.	9.	10.
11.	12.	13.	14.	15.
16.	17.	18.	19.	20.

Total:

(1) A parking ramp follows the curve in space traced by the function

$$r(t) = (\cos(t), \sin(t), t)$$

for $0 < t < 4\pi$. What is the curvature function $\kappa(t)$ of the ramp?

(A) $\kappa(t) = 2\sin(t)$
(B) $\kappa(t) = -1/\cos(t)$
(C) $\kappa(t) = 1/2$
(D) $\kappa(t) = 1$
(E) $\kappa(t) = \cos(t) + \sin(t)$
(F) none of the above

Answer to 1:

(2) An FBI agent puts a wire into the office of a Chicago politician which follows the curve

$$r(t) = \left(2\,t, \frac{4}{3}\,t^{3/2}, \frac{1}{2}\,t^2\right) \qquad 0 \le t \le 2.$$

How much wire does he need, i.e. what is the length of the curve above?

(A) 6

(B) 2

(C) 10

(D) 3

(E) 1

(F) none of the above

Answer to 2:

(3) An Iraqi reporter throws his shoe at President Bush during the President's last press conference in Bagdad.

The reporter is a horizontal distance of 12 feet from the President, and launches his shoe from a height of 3 feet. At lift-off, the shoe has a speed of 24 feet per second, and its trajectory makes an angle of 45° with the horizontal. The only force acting on the shoe during its flight is gravity, which produces a downward acceleration of 32 feet/sec^2.

How high will the shoe be above the ground when it is over the position where the President is standing? (You should treat the shoe as a point object; ignore the fact that it actually was a size 12 tasselled loafer.)

(A) 6 feet

(B) 5 feet

(C) 3 feet

(D) 8 feet

(E) 7 feet

(F) none of the above.

Answer to 3:

(4) The FBI embeds a microphone in the cylindrical birthday cake of a prominent politician. The base of the cake is the unit disk centered about the origin in the x-y plane, and the cake rises to height 1 up the z-axis. The position (x, y, z) of the microphone in the cake has probability density function

$$f(x,y,z) = \begin{cases} 4(x^2+y^2)z/\pi & \text{if } x^2+y^2 \le 1 \text{ and } 0 \le z \le 1 \\ 0 & \text{otherwise} \end{cases}$$

What is the probability that the microphone lies in the top half of the cake? (i.e., the z coordinate of the microphone is at least 1/2)?

(A) 1/4

(B) 1/2

(C) 2/3

(D) 3/4

(E) none of above

Answer to 4:

(5) Find

$$\int_{y=0}^{y=\sqrt{\pi}} \int_{x=y}^{x=\sqrt{\pi}} \sin(x^2)\ dx\ dy$$

by interchanging the order of integration.

(A) $\dfrac{\cos(1)}{2}$

(B) 1

(C) $\sin(1)$

(D) π

(E) none of the above

Answer to 5:

(6) A spherical star has center at $(0,0,0)$ and has radius 1 unit. The density of the star a position (x,y,z) is

$$f(x,y,z) = \begin{cases} (x^2+y^2+z^2)^{-1/2} & \text{if} \quad x^2+y^2+z^2 \leq 1 \\ 0 & \text{if} \quad x^2+y^2+z^2 > 1 \end{cases}$$

How much mass is there within a distance of a $1/2$ unit from the center of the star?

(A) $1/2$

(B) $\pi/3$

(C) $\pi/2$

(D) $\pi/4$

(E) none of above

Answer to 6:

(7) Let $T(x,y) = x^2 + y^2 - x - y$ be the temperature at the point (x,y) in the plane. A lizard sitting at the point $(1,3)$ wants to increase his surrounding temperature as quickly as possible. In which direction should he move?

(A) $\langle 1,1 \rangle$	(B) $\langle 1,3 \rangle$	(C) $\langle 1,5 \rangle$
(D) $\langle 1,7 \rangle$	(E) He should stay still.	(F) none of the above

Answer to 7:

(8) Let C be a smooth plane curve that starts at $(0,0)$ and finishes at $(\pi, 1/2)$, and let $\mathbf{F}$ be the vector field defined by

$$\mathbf{F}(x,y) = \langle y\cos(xy),\ x\cos(xy) + 2\rangle\,.$$

Compute the following line integral, or indicate that it cannot be determined from the above information:

$$\int_C \mathbf{F}\cdot d\mathbf{r}.$$

(A) -2

(B) -1

(C) 0

(D) 1

(E) 2

(F) cannot be determined from this information

Answer to 8:

(9) Let C be the plane curve which starts at $(-1,0)$, goes in a straight line to $(1,0)$, and then moves along the upper half of the circle $x^2+y^2=1$ back to the initial point. Compute the integral:

$$\oint_C (-y^3+x^2)\,dx + (x^3+y^3)\,dy.$$

(A) 0

(B) $\dfrac{\pi}{4}$

(C) $\dfrac{\pi}{2}$

(D) $\dfrac{3\pi}{4}$

(E) 2π

(F) none of the above

Answer to 9:

(10) Let M be the maximum value (and let m be the minimum value) of $f(x, y) = xy$ on the ellipse $4x^2 + y^2 = 8$. What is the value $M - m$?

(A) 2

(B) 3

(C) 4

(D) 5

(E) 6

(F) none of the above

Answer to 10:

(11) A biscuit in the shape of a cylinder has been put in an oven to bake. The radius and height of the biscuit are increasing at a rate of 0.2 and 0.1 cm per minute, respectively. How fast is the volume of the biscuit increasing when it has radius 5 cm and height 2 cm? *(in cm^3 per minute)*

(A) 4.5π

(B) 5π

(C) 5.5π

(D) 6π

(E) 6.5π

(F) none of the above

Answer to 11:

(12) At what point does the tangent plane of the surface

$$z^2 + xz + y^2 = 3$$

at $(1, 1, 1)$ intersect the z-axis?

(A) $(0, 0, 0)$

(B) $(0, 0, 1)$

(C) $(0, 0, 2)$

(D) $(0, 0, 3)$

(E) $(0, 0, 4)$

(F) none of the above

Answer to 12:

(13) The function $f(x,y) = x^3 + xy + y^3$ has two critical points, one of which is a local extremum P and one of which is wa saddle point Q. What is $f(P) - f(Q)$?

(A) $-\dfrac{1}{27}$

(B) $-\dfrac{1}{9}$

(C) $\dfrac{1}{3}$

(D) $\dfrac{1}{9}$

(E) $\dfrac{1}{27}$

(F) none of the above

Answer to 13:

(14) Suppose $y(x)$ is a solution to the initial value problem $y(0) = 10$ and

$$\frac{dy}{dx} = y + 2xy.$$

What is $y(2)$?

(A) $5e^{-4}$

(B) ∞

(C) 10

(D) 1

(E) $10e^{6}$

(F) none of the above

Answer to 14:

(15) Which of the following equations in x and y is equivalent to the statement that the vectors

$$A = \langle x+y, 1, y\rangle \quad \text{and} \quad B = \langle 1, x-y, -1\rangle$$

are perpendicular to each other?

(A) $2x - y = 0$

(B) $x - 2y = 0$

(C) $x - y = 0$

(D) $2x + y = 0-$

(E) $x + 2y = 0$

(F) none of the above

Answer to 15:

(16) Find an equation for the set of all points (x, y, z) in $\mathbb{R}^3$ which are equidistant from the points $A = (1, 0, 2)$ and $B = (-1, 2, -2)$.

(A) $-2x + 2(y + 1) - 4z = 0$

(B) $2x + 2(y - 1) - 4z = 0$

(C) $-2x + 2(y - 1) + 4z = 0$

(D) $2x + 2(y - 1) + 4z = 0$

(E) $-2x + 2(y - 1) - 4z = 0$

(F) none of the above

Answer to 16:

(17) Let $\mathbf{v} = \langle 0, 7, 0 \rangle$ and let $\mathbf{u}$ be a vector of length 5 which starts at the origin and lies in the x-y plane. Find the maximum value of the length of the vector $|\mathbf{u} \times \mathbf{v}|$.

(A) $|\mathbf{u} \times \mathbf{v}| = 12$

(B) $|\mathbf{u} \times \mathbf{v}| = 35$

(C) $|\mathbf{u} \times \mathbf{v}| = 30$

(D) $|\mathbf{u} \times \mathbf{v}| = 1$

(E) $|\mathbf{u} \times \mathbf{v}| = 140$

(F) none of the above

Answer to 17:

(18) The securities and exchange commission has determined that the amount $f(t)$ of fraud committed by a hedge fund manager at time t satisfies the differential equation

$$\frac{df}{dt} = f(t) \cdot \left(1 - \frac{f(t)}{100}\right)$$

in units of billions of dollars. They know that the initial value of f was $f(0) = 1$, and that at the current time t_0 the fraud is $f(t_0) = 50$. What is t_0?

(A) $t_0 = 99$

(B) $t_0 = \ln(99)$

(C) $t_0 = 100$

(D) $t_0 = \ln(100)$

(E) $t_0 = 50$

(F) none of the above

Answer to 18:

(19) Suppose that $y(x)$ is a solution of the initial value problem

$$2xy'(x) + y(x) = 6x \quad \text{for} \quad x > 0 \quad \text{and} \quad y(1) = 4.$$

What is $y(4)$?

(A) $y(4) = 4$

(B) $y(4) = 5$

(C) $y(4) = 3$

(D) $y(4) = 2$

(E) $y(4) = 9$

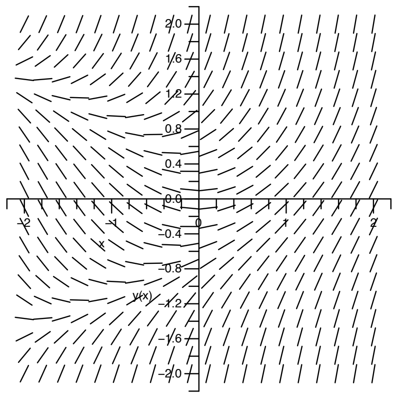
(F) none of the above

Answer to 19:

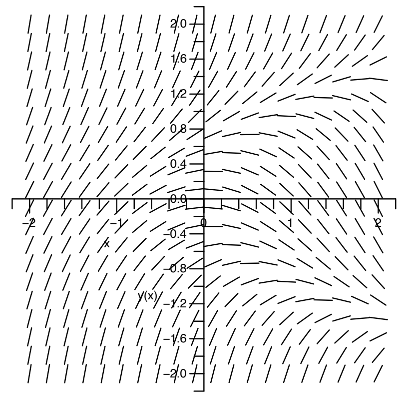
(20) Find which of the following direction fields corresponds to the differential equation

$$y' = y^2 - x$$

(A)



(B)



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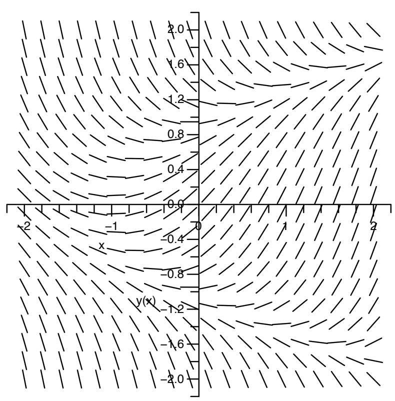

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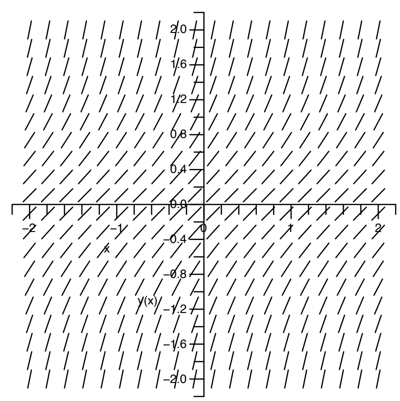

Answer to 20: