Problems for Final Exam
Multiple Choice

1. The volume of the region cut from the solid upper hemisphere $\rho \le 2,\ z \ge 0$ by the cone $\phi = \pi/6,\ z \ge 0$ is

(a) 1
(b) 2
(c) $\pi/2$
(d) $\pi/3$
(e) $\frac{16\pi}{3}(1 - \frac{\sqrt{3}}{2})$
(f) $\frac{8\pi}{6}\sqrt{3}$
(g) $1 - \frac{\sqrt{2}}{3}$
(h) $2 - \frac{\sqrt{2}}{3}$

Answer: (e)

2A. The principal unit normal $\mathbf{N}$ for the curve

$$r(t) = (2t+3)\mathbf{i} + (t^2 - 1)\mathbf{j}$$

at $t = 1$ is

(a) $\mathbf{i}/\sqrt{2} + \mathbf{j}/\sqrt{2}$
(b) $\mathbf{i}/\sqrt{2} - \mathbf{j}/\sqrt{2}$
(c) $-\mathbf{i}/\sqrt{2} - \mathbf{j}/\sqrt{2}$
(d) $-\mathbf{i}/\sqrt{2} + \mathbf{j}\sqrt{2}$
(e) $-\mathbf{i}$
(f) $\mathbf{i}$
(g) $-\mathbf{j}$
(h) $\mathbf{j}$

Answer: (c)

2B. A shell is fired out over the water from the top of a 160-ft. high cliff on the shoreline, at an angle of 30° with the horizontal, at an initial velocity of 320 ft./sec. Ignoring all forces except for gravity, when will it hit the water?

(a)	after $5+\sqrt{35}$ seconds	(b)	after $6+\sqrt{35}$ seconds
(c)	after $5+\sqrt{45}$ seconds	(d)	after $6+\sqrt{45}$ seconds
(e)	after $5+\sqrt{55}$ seconds	(f)	after $6+\sqrt{55}$ seconds
(g)	after $5+\sqrt{65}$ seconds	(h)	after $6+\sqrt{65}$ seconds

3. The length of the curve

$$r(t) = \cos^3 t\mathbf{j} + \sin^3 t\mathbf{k}, \quad 0 \le t \le \pi/2$$

is

(a) 1/2
(b) 3/4
(c) 1
(d) 5/4
(e) 3/2
(f) 7/4
(g) 2
(h) 9/4

Answer: (e)

4. The area in the first quadrant between the polar curves $r = 1+\cos\theta$ and $r = 1-\cos\theta$ is

(a) $\pi/4$
(b) 1
(c) 3/2
(d) $\pi/2$
(e) 7/4
(f) 2
(g) $3\pi/4$
(h) 5/2

Answer: (f)

5. In the Maclaurin series solution to the differential equation

$$y'' - xy = 0, \quad y(0) = 0,\ y'(0) = 1,$$

the coefficient of x^4 is

(a)	1	(b)	1/2
(c)	1/3	(d)	1/4
(e)	1/6	(f)	1/12
(g)	1/24	(h)	1/48

Answer: (f)

6. Let $f(x,y) = \sqrt{xy}$. Using differentials, a good approximation to $f(2.01, 1.98)$ is

(a)	1.98	(b)	1.985
(c)	1.99	(d)	1.995
(e)	2	(f)	2.005
(g)	2.01	(h)	2.015

Answer: (d)

Free Response

1. Find all fourth roots of $-1 + \sqrt{3}i$.

2. Find the general solution to

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 6\sin(2x).$$

3. Find the closest point to the origin which is on the intersection of the two planes

$$\begin{aligned} x + 2y + 3z &= 6 \quad \text{and} \\ x + 3y + 9z &= 9. \end{aligned}$$