MATH 114 Final Exam, Fall 2004

NAME:

Student ID number:

TA and section:

problem	score
1	
2	
3	
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5	
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8	
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10	
Free 1	
Free 2	
Free 3	
Free 4	

Multiple choice questions [5 points each]. Statistically, you will be neither rewarded nor penalized for guessing. Please show your work: partial credit will be possible and the correct answer will not guarantee full credit.

1. The area in the first quadrant between the polar curves $r = 1 + \cos\theta$ and $r = 1 - \cos\theta$ is

   (a) $\pi/4$

   (b) $\pi/2$

   (c) $7/4$

   (d) 2

   (e) $3\pi/4$

2. In the Maclaurin series solution to the differential equation

$$y'' - xy = 0, \quad y(0) = 0,\ y'(0) = 1\,,$$

   the coefficient of x^4 is

   (a) 1

   (b) $1/2$

   (c) $1/3$

   (d) $1/6$

   (e) $1/12$

3. The length of the curve

$$r(t) = \cos^3 t\mathbf{j} + \sin^3 t\mathbf{k}, \quad 0 \le t \le \pi/2$$

is

(a) 1
(b) 5/4
(c) 3/2
(d) 7/4
(e) 9/4

4. The country of Syldavia has a large stash of gold and by law, each citizen owns an equal share of this. As a result, the rate of immigration into Syldavia is proportional to the individual share of this wealth. From January 1, 2004 to January 1, 2005 the population exactly doubles. By approximately what date will it double again? (Assume it is OK to approximate popuation by a non-integer variable, and that birth, death and emmigration can be ignored.)

(a) July 1, 2005
(b) January 1, 2006
(c) January 1, 2007
(d) January 1, 2009
(e) January 1, 2020

5. If y is the solution to

$$\frac{dy}{dt} + 2ty = t \; ; \qquad\qquad y(0) = \frac{1}{2}$$

then $y(1)$ is equal to

(a) $\frac{1}{2} + \frac{1}{e}$

(b) $\frac{1}{2}$

(c) $\frac{1}{2} - \frac{1}{e}$

(d) $\frac{1}{e}$

(e) $\frac{1}{2} + \frac{2}{e}$

6. The position of a moving object is given by $\mathbf{r}(t) = \langle t^2, e^t, e^t + t^2 \rangle$. Please mark each of the four statements either true or false; no justification is needed.

T F The velocity vector, $\mathbf{v}$, is constant.

T F The curvature, κ, is always zero.

T F The binormal, $\mathbf{B}$, is always in the direction $\langle 1, 1, -1 \rangle$.

T F The torsion, τ, is always zero.

7. Let B be the solid unit ball. Compute

$$\int\int\int_B \exp((x^2+y^2+z^2)^{3/2})\,dV\,.$$

(a) $\frac{4\pi}{3}(e^{5/2}-1)$

(b) $\frac{4\pi}{3}(e-1)$

(c) $\frac{\pi}{3}(4e^{3/2}-1)$

(d) $\frac{\pi}{3}(8e^{1/2}-4)$

(e) $\frac{2\pi}{3}(e^{5/2}-e)$

8. The function $y(x)$ solves the differential equation

$$\frac{dy}{dx}=f(x,y)$$

with initial conditions $y(x_0)=y_0$. If we know that $f(x_0,y_0)=1$, $f_x(x_0,y_0)=2$, $f_y(x_0,y_0)=3$, $f_{xx}(x_0,y_0)=4$, $f_{xy}(x_0,y_0)=5$ and $f_{yy}(x_0,y_0)=6$, then the value of d^y/dx^2 at (x_0,y_0) is:

(a) 1

(b) 4

(c) 5

(d) 6

(e) 15

9. Reversing the order of integration transforms

$$\int_{-2}^{0}\int_{2x+4}^{4-x^2} f(x,y)dydx$$

into

(a) $\int_{2x+4}^{4-x^2}\int_{-2}^{0} f(x,y)dxdy$ (b) $\int_0^4\int_{\sqrt{4-y}}^{\frac{y-4}{2}} f(x,y)dxdy$

(c) $\int_0^4\int_{-2}^{0} f(x,y)dxdy$ (d) $\int_0^4\int_{\frac{y-4}{2}}^{\sqrt{4-y}} f(x,y)dxdy$

(e) $\int_{-2}^{0}\int_{2y+4}^{4-y^2} f(x,y)dxdy$ (f) $\int_0^4\int_{-\sqrt{4-y}}^{\frac{y-4}{2}} f(x,y)dxdy$

10. Let C be the curve of intersection of the cylinder $\frac{x^2}{25}+\frac{y^2}{9}=1$ with the plane $3z=4y$. Let L be the line tangent to C at the point $(0,-3,-4)$. What is the x-coordinate of the point of intersection of L and the plane $2x-3y-4z=27$?

(a) -2

(b) 1

(c) 2

(d) 3

(e) 5

Free response questions. Answer the following questions as completely as you can. Show your work and clearly indicate your final answers.

1. [**6 points**] Find all fourth roots of $-1 + \sqrt{3}i$.

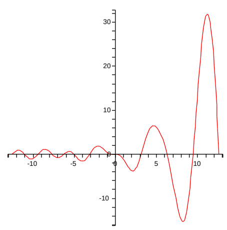
2. [**7 points**] A shell is fired out over the water from the top of a 160-ft. high cliff on the shoreline, at an angle of $30°$ with the horizontal, at an initial velocity of 320 ft./sec. Ignoring all forces except for gravity, when will it hit the water?

3. [**7 points**] Find the general solution to

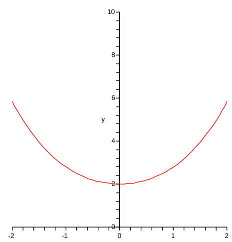
$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 6\sin(2x)\,.$$

4. [**10 points**] Match the differential equations below to the graph that represents a solution to that differential equation. Explain your reasoning.

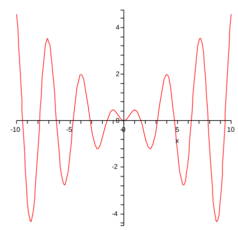
Graphs:



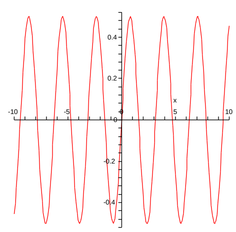
(i)



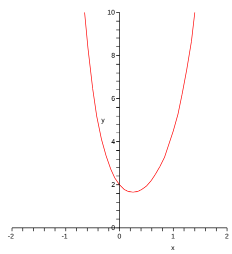
(ii)



(iii)



(iv)



(v)

Equations:

(a) $4y'' - 3y = 0$

(b) $y' + 3y = 3e^{2t}$

(c) $y'' + 4y = 2cos(2x)$

(d) $y'' + 4y = 0$

(e) $16y'' - 8y' + 17y = -16\cos(2x) - 47\sin(2x)$

Answers:

Equations	(a)	(b)	(c)	(d)	(e)
Graphs					