Math 114, FINAL EXAM
December 18, 2003

INSTRUCTIONS	OFFICIAL USE ONLY

INSTRUCTIONS:

1. Please complete the information requested below. There are 14 multiple choice problems, 6 open answer problems. No partial credit will be given on the multiple choice questions.
2. Please show all your work on the exam itself. Correct answers with little or no supporting work will not be given credit.
3. Only 18 of the 20 problems on the exam will count towards your final grade. We will automatically drop the worst score answer from each part (multiple choice or open answer) of the exam.
4. You are allowed to use one hand-written sheet of paper with formulas. *No calculators, books or other aids are allowed.* Please turn in your crib sheet together with your exam.

- **Name** (please print):
- **Name of your professor:**
  ◯ **Dr. Drumm** ◯ **Dr. Kong** ◯ **Dr. Pantev**
- **Name of your TA:** ◯ **M.Can** ◯ **M.Cross** ◯ **T.Jaeger** ◯ **J.Long** ◯ **M.Sabitova** ◯ **T.Zhu**
- **I certify that all of the work on this test is my own.**
  Signature:
- **Recitation section:**

Problem	Points
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Total	

Part I: Multiple choice questions

1. What is the x-intercept of the line tangent to the curve $x(t) = 3+\cos(\pi t)$, $y(t) = t^2+t+1$ when $t = 1$?

(A) 3 (B) 2 (C) does not exist

(D) 1 (E) $\frac{2}{3}$ (F) $\frac{5}{4}$

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2. Evaluate

$$\int_0^1 \int_{\sqrt{x}}^1 3e^{y^3}\,dydx$$

(A) e (B) $e-1$ (C) $e+1$

(D) $3e$ (E) $3e-2$ (F) none of the above

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3. Which of these differential equations is exact as written?

(I) $(x+y^2)\,dx + (2xy+y^3)\,dy = 0$

(II) $\left(x+\dfrac{1}{y}\right)\,dx + \left(y+\dfrac{1}{x}\right)\,dy = 0$

(III) $\left(1-\dfrac{y}{x^2}\right)\,dx + \left(y+\dfrac{1}{x}\right)\,dy = 0$

(A) (I) only (B) (II) only (C) (III) only

(D) (I) and (II) (E) (I) and (III) (F) (II) and (III)

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4. Which of the sets described by the spherical coordinate inequalities are bounded?

(I) $\rho = 1,\ \varphi < \dfrac{\pi}{2}$

(II) $\rho + \theta^2 < 1$

(III) $\theta + \varphi^2 < \pi$

(A) (I) only (B) (II) only (C) (III) only

(D) (I) and (II) (E) (I) and (III) (F) (II) and (III)

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5. The function $y(x)$ satisfies $y'' - 2y' + y = 0$. The graph of $y(x)$ passes through the origin and satisfies $y'(0) = 1$. What is $y''(1)$?

(A) e (B) $2e$ (C) $3e$ (D) $4e$ (E) $5e$ (F) $6e$

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6. Find the cosine of the angle between the vectors $2\widehat{\mathbf{\imath}} + 2\widehat{\mathbf{\jmath}} + \widehat{\mathbf{k}}$ and $2\widehat{\mathbf{\imath}} + \widehat{\mathbf{\jmath}} - 2\widehat{\mathbf{k}}$.

(A) $\dfrac{2}{9}$ (B) $\dfrac{1}{3}$ (C) $\dfrac{4}{9}$ (D) $\dfrac{5}{9}$ (E) $\dfrac{2}{3}$ (F) $\dfrac{4}{3}$

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7. Find the area inside the curve $r = \dfrac{2}{3}\cos(\theta),\ -\dfrac{\pi}{2} \le \theta \le \dfrac{\pi}{2}$.

(A) $\dfrac{4}{3}$ (B) $\dfrac{2\pi}{3}$ (C) $\dfrac{1}{3\sqrt{3}}$ (D) $\dfrac{2}{9}$ (E) $\dfrac{1}{\sqrt{3}}$ (F) $\dfrac{\pi}{9}$

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8. Compute the product $(1+\sqrt{3}i)^6(1+i)^8$.

(A) 2^6 (B) 2^7 (C) 2^8 (D) 2^9 (E) 2^{10} (F) 2^{11}

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9. Find the arclength of the parameterized path

$$x(t) = \frac{t^2}{2}$$
$$y(t) = \frac{t^3}{3}$$

for $1 \le t \le 3$

(A) $\sqrt{10}$ (B) $\dfrac{\sqrt{10}}{3}$ (C) $\sqrt{10}+\sqrt{2}$

(D) $\dfrac{\sqrt{10}-\sqrt{2}}{3}$ (E) $\dfrac{10\sqrt{10}-2\sqrt{2}}{3}$ (F) none of the above

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10. Use polar coordinates to evaluate the integral

$$\int\int_D \sqrt{2-x^2-y^2}\,dxdy,$$

where $D = \{(x,y)|x^2+y^2 \le 1, y \ge x\}$.

(A) 0 (B) -1 (C) $\pi(1-2\sqrt{2})$

(D) $\dfrac{\pi}{\sqrt{2}}$ (E) $\dfrac{\pi}{6}(8-2\sqrt{2})$ (F) $\dfrac{\pi}{3}(2\sqrt{2}-1)$

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11. Use the linear approximation of the function $e^{x\cos y}$ at $(0,0)$ to estimate $e^{(0.1)\cdot\cos(-0.2)}$.

(A) -0.1 (B) -0 (C) 0.1 (D) 0.3 (E) 1 (F) 1.1

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12. The function $f(x, y) = x^3 + 6xy - y^3$ has

(A) two saddle points
(B) one saddle point and one local maximum
(C) one saddle point and one local minimum
(D) one local maximum and one local minimum
(E) two local maxima
(F) none of the above

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13. Find the point of intersection of the line $x = 1 + 2t$, $y = 3t$, $z = t - 1$ and the plane tangent to $x^2 + 2y = z^2$ at $(2, 0, -2)$.

(A) $(3, 3, 0)$ (B) $\left(0, -\frac{3}{2}, -\frac{3}{2}\right)$ (C) $(1, 0, -1)$

(D) $(-1, -3, -2)$ (E) $\left(\frac{1}{2}, \frac{1}{6}, -\frac{1}{2}\right)$ (F) $(7, 9, 2)$

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14. Which of the following unit vectors is in the direction of fastest decrease of $f(x, y, z) = xe^{yz}$ at the point $(1, 0, 1)$?

(A) $\frac{2}{\sqrt{5}}\hat{\imath} + \frac{1}{\sqrt{5}}\hat{\jmath}$ (B) $\frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath}$ (C) $\frac{1}{\sqrt{2}}\hat{\imath} - \frac{1}{\sqrt{2}}\hat{\jmath}$

(D) $\frac{1}{\sqrt{5}}\hat{\imath} + \frac{2}{\sqrt{5}}\hat{\jmath}$ (E) $\frac{1}{\sqrt{5}}\hat{\imath} - \frac{2}{\sqrt{5}}\hat{\jmath}$ (F) none of the above

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PART II BEGINS ON THE NEXT PAGE

Part II: Open answer questions

15. True or false. Explain your reasoning.

(a) The surface given in cylindrical coordinates by the equation $r = \pi z \csc(\theta)$ is a plane.

(b) The spherical equations $\phi = \pi/3$, $\theta = \pi/4$ describe a single ray.

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16. Find the maximum of the function $f(x, y) = x^2 - 2y^2$ on the set $x^2 + 2y^2 \leq 4$.

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17. Find the volume of the wedge shaped solid that lies above the xy-plane, below the plane $z = x$ and within the cylinder $x^2 + y^2 = 4$.

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18. Match the polar coordinate equations to their graphs, for $0 \leq \theta \leq 2\pi$, $-\infty < r < \infty$. (**Hint:** it is easier to use the polar equations directly than to convert to Cartesian equations.) Enter your answers in the table below. Give a reason for each choice.

Equations:

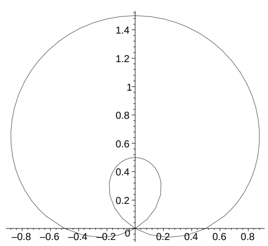
(i) $r = 4\cos(2\theta)$

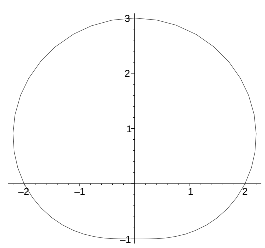
(ii) $r = 2 + \sin(\theta)$

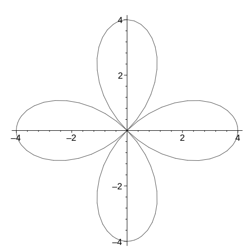
(iii) $r^2 = \sin(\theta)$

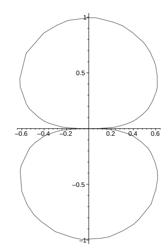
(iv) $2r = 1 + 2\sin(\theta)$

Graphs:

(a) 

(b) 

(c) 

(d) 

Answers:

Graphs	(a)	(b)	(c)	(d)
Equations				

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19. Let A, B and C be the vertices of a triangle in the plane and let a, b and c be, respectively, the midpoints of the opposite sides.Show that

$$\overrightarrow{Aa} + \overrightarrow{Bb} + \overrightarrow{Cc} = \overrightarrow{0}.$$

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20. Evaluate the limit

$$\lim_{(x,y)\to(0,0)} \frac{x^3y^3 - x^2 - y^2}{x^2 + y^2}$$

if it exists.
