Math 114, MAKEUP FINAL EXAM

January 14, 2004

INSTRUCTIONS	OFFICIAL USE ONLY

INSTRUCTIONS:

1. Please complete the information requested below. There are 14 multiple choice problems, 6 open answer problems. No partial credit will be given on the multiple choice questions.
2. Please show all your work on the exam itself. Correct answers with little or no supporting work will not be given credit.
3. Only 18 of the 20 problems on the exam will count towards your final grade. We will automatically drop the worst score answer from each part (multiple choice or open answer) of the exam.
4. You are allowed to use one hand-written sheet of paper with formulas. *No calculators, books or other aids are allowed.* Please turn in your crib sheet together with your exam.

• **Name** (please print):

• **Name of your professor:**
◯ **Dr. Drumm** ◯ **Dr. Kong** ◯ **Dr. Pantev**

• **Name of your TA:** ◯ **M.Can** ◯ **M.Cross** ◯ **T.Jaeger** ◯ **J.Long** ◯ **M.Sabitova** ◯ **T.Zhu**

• **I certify that all of the work on this test is my own.**
Signature:

• **Recitation section:**

Problem	Points
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20.	
Total	

Part I: Multiple choice questions

1. Find the y-intercept of the line tangent to the parameterized curve $x(\theta) = \cos(\theta)/2$, $y(t) = 4\sin(\theta)$ when $\theta = \pi/4$?

(A) $\sqrt{2}$ (B) $-2\sqrt{2}$ (C) 2 (D) $4\sqrt{2}$ (E) 4 (F) 0

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2. Evaluate

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \frac{4}{1+x^2+y^2} dydx$$

(A) $\pi \ln(2)$ (B) $\pi \ln(3)$ (C) $\pi \ln(4)$

(D) $\pi \ln(5)$ (E) $\pi \ln(6)$ (F) none of the above

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3. Evaluate the limit

$$\lim_{(x,y)\to(0,0)} \frac{x^3 - xy^2}{x^2+y^2}.$$

(A) 1 (B) 0 (C) $-\infty$ (D) -1 (E) $\frac{1}{2}$ (F) does not exist

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4. Which of the sets described by the spherical coordinate inequalities are bounded?

(I) $\rho \geq 1$, $\varphi = \frac{\pi}{2}$

(II) $\rho \cdot \theta^2 < 1$

(III) $\theta^2 + \varphi^2 < \pi$

(A) (I) only (B) (II) only (C) (III) only

(D) (I) and (II) (E) (I) and (III) (F) none

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5. The function $y(x)$ solves the initial value problem

$$2x + y^3 + 3xy^2 \frac{dy}{dx} = 0, \qquad y(1) = 2.$$

What is $y(3)$?

(A) $-\sqrt[3]{18}$ (B) 0 (C) $\sqrt[3]{17}$

(D) $\sqrt{17}$ (E) 17 (F) none of the above

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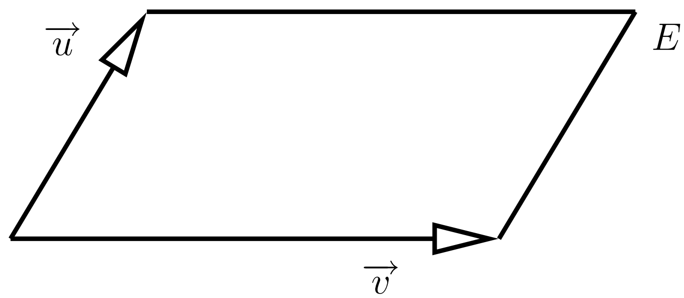
6. Let E be a parallelogram in the plane, defined by two vectors $\overrightarrow{u}$, $\overrightarrow{v}$:



Compute the ratio:

$$\frac{\text{length of the cross-product of the diagonals of } E}{\text{area of } E}$$

(A) 2 (B) 0 (C) undetermined (D) $\frac{1}{2}$ (E) $\frac{1}{3}$ (F) $|\overrightarrow{u}|^2 \cdot |\overrightarrow{v}|^2$

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7. Find the area inside one leaf of the curve $r^2 = \sin(4\theta)$.

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$

(D) $\frac{1}{5}$ (E) $\frac{1}{6}$ (F) none of the above

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8. The plane containing the parallel lines

$$L_1 : \left| \begin{array}{rcl} x-2 & = & t \\ y-1 & = & 2t \\ z-2 & = & t \end{array} \right. \qquad \text{and} \qquad L_2 : \left| \begin{array}{rcl} x+2 & = & t \\ y & = & 2t \\ z-2 & = & t \end{array} \right.$$

intersects the y-axis at the point

(A) $y = 0$ (B) $y = -2$ (C) $y = 4$

(D) $y = 2$ (E) $y = -3$ (F) none of the above

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9. Find the length of the curve

$$r = \frac{1}{\pi} \cdot \sqrt{1 + \sin(2\theta)}, \qquad 0 \le \theta \le \frac{\pi}{4}.$$

(A) $\frac{\pi}{2}$ (B) 1 (C) $\frac{1}{2\sqrt{2}}$ (D) $\frac{\pi}{\sqrt{2}}$ (E) $\frac{1}{4}$ (F) 2

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10. Evaluate the integral

$$\int\int_D \frac{2xy}{x^2+y^2} dxdy,$$

where D consists of all points in the second quadrant which are also contained in the annulus $1 \le x^2 + y^2 \le 2$.

(A) $\frac{1}{2}$ (B) 1 (C) 0 (D) $-\frac{3}{2}$ (E) -1 (F) none of the above

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11. Use the linear approximation of the function $f(x,y) = \tan(x\ln(1+y))$ at $(\pi, 0)$ to estimate $f(\pi - 0.25, 0.1)$.

(A) -0.1π (B) 1.25π (C) 0.75π (D) 0.1π (E) 0.35 (F) -0.2

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12. The function $f(x,y) = 8x^3 + y^3 + 6xy$ has

(A) one local maximum and one local minimum
(B) one saddle point and one local maximum
(C) one saddle point and one local minimum
(D) two saddle points
(E) two local maxima
(F) two local minima

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13. Find the volume of the solid inside the cylinder $x^2 + y^2 = 1$, above the xy-plane and below the plane $z = x + 1$.

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) π (D) $\pi + \frac{1}{3}$ (E) 2π (F) $2\pi + \frac{2}{3}$

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14. Compute the partial derivative $\frac{\partial w}{\partial r}$ at $r = \pi$, $s = 0$ for

$$w = \sin(3x - 2y), \quad x = r + \sin(s), \quad y = rs^2.$$

(A) -3 (B) -2 (C) -1 (D) 1 (E) 2 (F) 3

PART II BEGINS ON THE NEXT PAGE

Part II: Open answer questions

15. True or false. Explain your reasoning.

(a) The surface given in cylindrical coordinates by the equation

$$r = z \sec\left(\theta + \frac{\pi}{4}\right)$$

is a plane.

(b) The spherical equations $|\phi| = \pi/3$, $|\theta| = \pi/4$ describe a pair of intersecting lines.

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16. Find the absolute maximum and the absolute minimum of the function $f(x, y) = 3x - 4y + 1$ on the curve $(x-1)^2 + (x-y)^2 = 25$.

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17. Find a line that goes through the point $(-1, 1, 0)$ and is perpendicular to the plane tangent to the surface $z = \sqrt{8 - x^2 - 3y^2}$ at $(-1, 1, 2)$.

(CONTINUED ON NEXT PAGE)

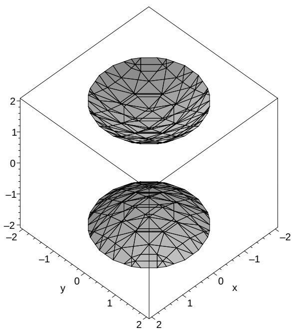
18. Match the equations to their graphs. Enter your answers in the table at the bottom of the page and show your reasoning on the next page.

Equations:

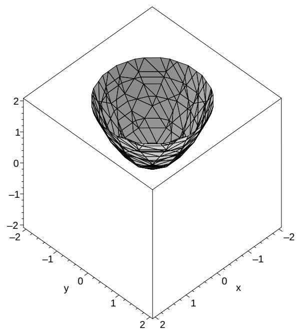
(1) $z - x^2 - y^2 = 0$ (2) $z^2 - x^2 - y^2 = 1$

(3) $\sin(x)y + \sin(y)x = 1$ (4) $z^2 - x^2 = 2$

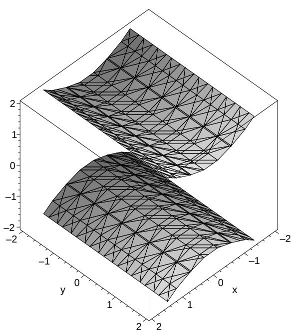
Graphs:



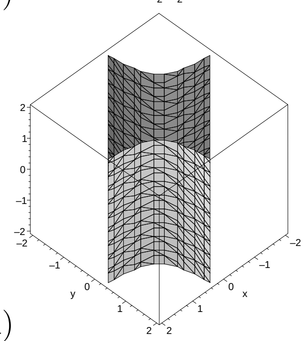
(a)



(b)



(c)



(d)

Answers:

Graphs	(a)	(b)	(c)	(d)
Equations				

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19. Find all (real and complex) roots of the equation $x^8 + 2x^4 + 1 = 0$.

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20. Show that $\rho(x) = x^{-3}$ is an integrating factor of

$$(x + 2y)\,dx - x\,dy = 0$$

and then solve the equation using the integrating factor.