MATH 114 Final Exam, Fall 2007

NAME:

Student ID number:

TA:

Section number or time:

Multiple choice questions [5 points each]. A score of +1 will be given if no answer is marked. No calculators are allowed on this exam. Please show all work that you wish to be considered for partial credit and circle the letter corresonding to your choice of answer.

1. The nearest that the curve $x^3 + y^3 = 1$ gets to the origin is a distance of

   (a) $1/2$

   (b) $1/\sqrt[3]{2}$

   (c) 1

   (d) $\sqrt[3]{2}$

   (e) there is no closest distance

2. A certain forced oscillator is modeled by the differential equation

$$my'' + \alpha y' + 12y = \cos(2t)$$

   where $m, \alpha \geq 0$. Suppose we observe a solution that oscillates with an amplitude that grows proportionally to t. What are m and α?

   (a) $m = 3, \alpha = -12$

   (b) $m = 3, \alpha = 0$

   (c) (m, α) are any pair satisfying $\alpha^2 = 48m$

   (d) (m, α) are any pair satisfying $4m + 2\alpha + 12 = 0$

   (e) No values of m and α will have such a solution

3. The derivatives of a function f in the directions $\mathbf{i}$, $\mathbf{i}+\mathbf{j}+\mathbf{k}$ and $\mathbf{i}+2\mathbf{j}+3\mathbf{k}$ respectively are 0, 0 and 1. What is the direction of fastest increase of f?

   (a) $\mathbf{j}-\mathbf{k}$

   (b) $\mathbf{k}-\mathbf{j}$

   (c) $\mathbf{i}+2\mathbf{j}+3\mathbf{k}$

   (d) $-\mathbf{i}+\mathbf{j}+2\mathbf{k}$

   (e) $\mathbf{j}+\mathbf{k}$

4. Find the volume of the solid in the half-space $z \geq 0$ that is enclosed by the unit sphere $x^2+y^2+z^2=1$ and the cone $z^2=3(x^2+y^2)$.

   (a) $\dfrac{(\sqrt{3}-1)\pi}{3}$

   (b) π

   (c) $\dfrac{(2-\sqrt{3})\pi}{3}$

   (d) $\dfrac{2\pi}{3}$

   (e) $\dfrac{\pi}{\sqrt{3}}$

5. The function $y(x)$ solves the differential equation

$$\frac{dy}{dx} = f(x, y)$$

with initial conditions $y(x_0) = y_0$. If we know that $f(x_0, y_0) = 5$, $f_x(x_0, y_0) = 4$ and $f_y(x_0, y_0) = 3$, then the value of d^2y/dx^2 at (x_0, y_0) is:

(a) 5

(b) 12

(c) 15

(d) 16

(e) 19

6. A gas inside a chamber is being created at the constant rate of 40 kilograms per hour. Due to a reaction (for example $AAA \to A_3$, though you don't need to understand the chemistry here) the gas disappears at a rate $5M^3$ kilograms per hour, where M is the number of kilograms of the gas present in the chamber. If the chamber begins with no gas in it, the amount of this gas after one day will be:

(a) less than 1 kilogram

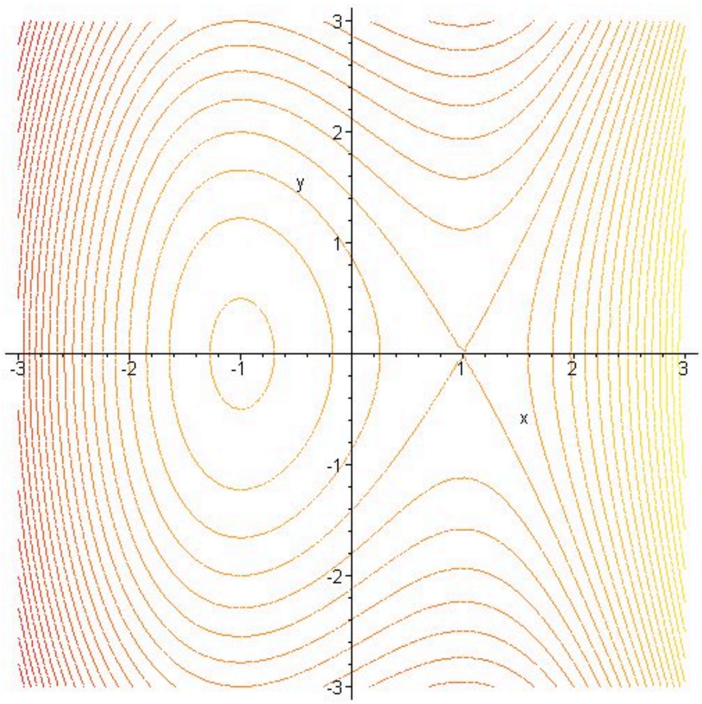
(b) between 1 and 1.9 kilograms

(c) between 1.9 and 2 kilograms

(d) between 2 and 7.9 kilograms

(e) more than 7.9 kilograms

7. Which one of the following statements could be true of the function $f(x,y)$ whose contour plot is shown?



(a) $f(x,y)$ has no critical points inside the square.

(b) $f(x,y)$ has a saddle at $(1,0)$ and a maximum at $(-1,0)$.

(c) $f(x,y)$ has a minimum at $(1,0)$ and a saddle at $(-1,0)$.

(d) $f(x,y)$ has a local maximum and a local minimum inside the square $-3 \le x, y \le 3$.

(e) $f(x,y)$ has saddles inside the square $-3 \le x, y \le 3$ but no local maxima or minima in the square.

8. Suppose $y'(t) = e^{y-t^2}$ with initial condition $y(0) = C$. Which one of the following is true?

(a) For all C, $y(t)$ increases to a finite limit as $t \to \infty$.

(b) For all C, y increases to infinity at some finite time $t > 0$.

(c) For all C, y increases to infinity as $t \to \infty$.

(d) If C is positive, then y increases to infinity as $t \to \infty$, while if C is negative, y approaches a finite limit as $t \to \infty$.

(e) Depending on C, y may increase to a finite limit, become infinite in finite time, or increase without bound as $t \to \infty$.

9. [**8 points**] Santa Claus's sleigh loses power after taking off at 64 feet per second from a 32 foot high roof at an angle of 45°. How many seconds elapse before he hits the ground? (For this problem, assume that Santa exists and that his magic is not powerful enough to alter the earth's gravitational acceleration of $32\, f/s^2$.)

Answer = ______________________________

10. [**8 points**] Find the general solution to $y' = 2 - y/t$.

Answer = ________________________________

11. [**8 points**] Use Euler's method with step size $1/2$ to approximate the value $y(1)$ if $y(t)$ solves the initial value problem

$$y' = 1 - \frac{1}{(1+t)(1+y)} \; ; \quad y(0) = 1.$$

Answer = ____________

12. [**3 points each part**] A ball travels on the unit circle in the xy-plane with θ given by $\theta(t) = (\pi/2)\cos(t)$.

   (a) Draw a picture showing the position of the ball and the directions of the normal and tangential components of its acceleration at a time when the particle is not on the x or y-axis. (You do not need to compute these exactly.)

   (b) List the values of t with $0 < t \leq 2\pi$ such that the acceleration of the ball is parallel to the x-axis.

times are: ________________________________

13. [**8 points**] Find the terms up through the t^5 term of a power series solution to the equation $y'' + 2ty = 0$ with initial conditions $y(0) = 1, y'(0) = 0$.

Answer = ________________________________