Name: ____

Midterm Exam III for Math 110, Spring 2015

April 23, 2015

Problem	Points	Score
1	12	
2	12	
3	16	
4	8	
5	12	
6	12	
7	12	
8	12	
9	12	
10	12	
Total	120	

- You have ninety minutes for this exam.
- Please show **ALL** your work on this exam paper. Partial credit will be awarded where appropriate.
- **CLEARLY** indicate final answers. Use words (doesn't have to be that many words) to connect mathematical formulas and equations.
- **NO** books, notes, laptops, cell phones, calculators, or any other electronic devices may be used during the exam. One 8.5×11 cheatsheet, handwritten is allowed; it may be double-sided.
- No form of cheating will be tolerated. You are expected to uphold the Code of Academic Integrity.

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this midterm examination.

Signature: ________________________________

Date: ________________________________

1. (a) Find the general solution to the differential equation

$$e^{kt}\frac{dz}{dt} = \frac{a}{z}\,.$$

(b) If the units of t are seconds and the units of z are feet, what are the units of k and a?

2. (a) Circle the number of the correct solution to the initial value problem

$$y' = y\frac{\sin x}{x^2}\ ; \quad y(2) = 3\,.$$

(i) $y = 3e^{2-x}$

(ii) $y = \ln\left(e^3 + \int_2^x \frac{\sin t}{t^2}\, dt\right)$

(iii) $y = e^{3+\int_2^x \frac{\sin t}{t^2}\, dt}$

(iv) $y = 3\int \frac{\sin x}{x^2}\, dx$

(v) $y = e^{\int \frac{\sin x}{x^2}\, dx + C}$

(vi) $y = 3 + e^{\int_2^x \frac{\sin t}{t^2}\, dt}$

(vii) $y = 3e^{\int_2^x \frac{\sin t}{t^2}\, dt}$

(b) Precisely one of the following statements is true; please circle the corresponding number.

(i) $y(x) \to \infty$ as $x \to \infty$.

(ii) $y(x)$ is defined for all $x \geq 2$ and has a horizontal asymptote.

(iii) $y(x)$ is defined for all $x \geq 2$ and has a vertical asymptote.

(iv) $y(x)$ has a vertical asymptote and is not defined past that point.

(v) The solution may have a vertical or horizontal asymptote, depending on the initial condition.

(vi) $y(x) \to -\infty$ as $x \to \infty$.

3. A benefactor sets up a trust fund by giving a steady stream of money, with the rate of giving increasing over time according to the formula "rate = $\$100,000t$ per year at time t years," starting at time $t = 0$ years with no money in the account. The account also grows by return on investment at the rate of 2% per year.

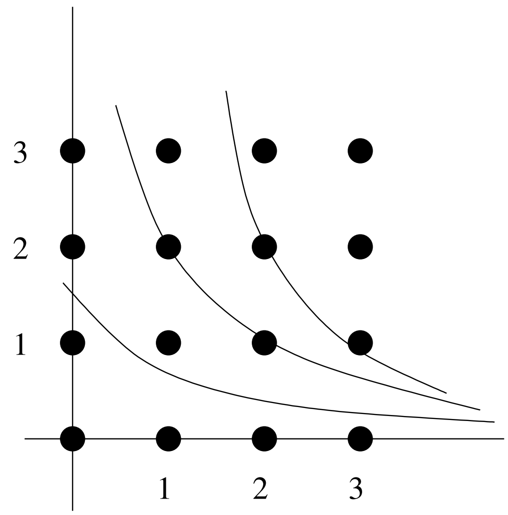
   (a) Write an initial value problem for the value after time t years.

   (b) Solve this initial value problem.

   (c) Write an exact expression for the value after one year.

   (d) Give an approximate numerical value by using the quadratic Taylor approximation to the expression computed in part (c).

4. A contour plot is shown for the function $u(x, y)$. For each statement, please circle T for true or F for false.



T/F: u is changing faster near $(1, 2)$ than it is near $(2, 1)$.

T/F: It is possible that $u(x, y)$ is $\dfrac{1}{1 + x + y}$.

T/F: If $u(x, y)$ represents my utility then I am indifferent between the outcomes at $(1, 2)$ and $(2, 1)$.

T/F: It is possible that $u(x, y) = ye^{-x}$.

5. Compute $\displaystyle\int_1^5 \int_0^2 \frac{x+1}{y}\, dx\, dy\,.$

6. Compute the average temperature over the upper half of the unit disk in the x-y plane if the temperature is given by $T(x, y) = 2y$.

Laying out the steps in a way we can follow will help in the event that we are evaluating your solution for partial credit.

7. A trapezoid T has corners at $(\pm 2, 0)$ and $(\pm 1, 3)$.

(a) Describe the region T in horizontal strips:

$$T = \{(x, y) \, : \, \cdots\} .$$

(b) If the dart is thrown at the region T and lands in a random location, distributed uniformly over T, what density describes this probability distribution?

(c) What is the mean of the Y value chosen by the dart?

8. Suppose that profit depends on price, cost per unit, and number sold according to the formula $P = n(p - c)$. The number sold is a function of the price.

   (a) Make a branch diagram for this.

   (b) State which of P, p, c and n are independent variables, which are dependent variables and which are intermediate variables.

   (c) Suppose that the present values of p, c and n are respectively 3, 1 and 10,000. Write a formula for the increase in profit per unit increase in price assuming that cost is held constant. Your formula may contain symbols for functions and derivatives not explicitly known.

9. A customer's satisfaction with her hotel room in Barbados is modeled by the utility function $u = 100 - (T - 70)^2 - \frac{c}{10}$ where T is temperature in degrees Farenheit and c is cost per night in dollars. If the room is presently $78°F$ and costs \$200 per night, how many more dollars per night would she be willing to pay per extra degree of lower temperature in the room?

10. Use the increment theorem for the function $f(x, y) = \sqrt{x + \ln y}$ to give a numerical estimate of $\sqrt{99 + \ln 1.3}$.

You don't need to state the theorem, but if you want to be eligible for partial credit you should state the values of Δx and Δy and show how the relevant partial derivatives are evaluated.

TABLE 8.1 Basic integration formulas

1. $\int k\,dx = kx + C$ (any number k)	12. $\int \tan x\,dx = \ln\lvert\sec x\rvert + C$
2. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$ $(n \neq -1)$	13. $\int \cot x\,dx = \ln\lvert\sin x\rvert + C$
3. $\int \frac{dx}{x} = \ln\lvert x\rvert + C$	14. $\int \sec x\,dx = \ln\lvert\sec x + \tan x\rvert + C$
4. $\int e^x\,dx = e^x + C$	15. $\int \csc x\,dx = -\ln\lvert\csc x + \cot x\rvert + C$
5. $\int a^x\,dx = \frac{a^x}{\ln a} + C$ $(a > 0,\ a \neq 1)$	16. $\int \sinh x\,dx = \cosh x + C$
6. $\int \sin x\,dx = -\cos x + C$	17. $\int \cosh x\,dx = \sinh x + C$
7. $\int \cos x\,dx = \sin x + C$	18. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$
8. $\int \sec^2 x\,dx = \tan x + C$	19. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$
9. $\int \csc^2 x\,dx = -\cot x + C$	20. $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\sec^{-1}\left\lvert\frac{x}{a}\right\rvert + C$
10. $\int \sec x \tan x\,dx = \sec x + C$	21. $\int \frac{dx}{\sqrt{a^2 + x^2}} = \sinh^{-1}\left(\frac{x}{a}\right) + C$ $(a >$
11. $\int \csc x \cot x\,dx = -\csc x + C$	22. $\int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1}\left(\frac{x}{a}\right) + C$ $(x >$