Name: _____ Section (circle one): 001 002

Midterm Exam III for Math 110, Fall 2014

December 2, 2014

Problem	Points	Score
1	12	
2	12	
3	12	
4	12	
5	12	
6	12	
7	12	
8	12	
9	12	
10	12	
Total	120	

- You have ninety minutes for this exam.
- Please show **ALL** your work on this exam paper. Partial credit will be awarded where appropriate.
- **CLEARLY** indicate final answers. Use words (doesn't have to be that many words) to connect mathematical formulas and equations.
- **NO** books, notes, laptops, cell phones, calculators, or any other electronic devices may be used during the exam. One 8.5×11 cheatsheet, handwritten is allowed; it may be double-sided.
- No form of cheating will be tolerated. You are expected to uphold the Code of Academic Integrity.

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this midterm examination.

Signature: _____

Date: _____

1. Find the general solution to the differential equation

$$y' = \frac{1}{\sqrt{xy}}\,.$$

2. Find the general solution to the differential equation

$$y' = 2x - y\,.$$

3. Solve the initial value problem

$$\frac{dy}{dx} = x - xy\,, \quad y(0) = 11\,.$$

4. A population grows by net births at an annual rate equal to 2% of the population and by net immigration at an annual rate of 10,000 people per year[1].

   (a) Write a differential equation for the population as a function of time.

   (b) Find the general solution to this equation.

[1] "net births" means births minus deaths, so you don't have to worry about the two separately; similarly, "net immigration" means immigration minus emmigration."

(c) Find the population at time $t = 10$ years if the population at time $t = 0$ is one million people. Leave this in the form of an exact expression; do not evaluate as a decimal number.

(d) Give an approximate value for the population at time $t = 10$ years by using the quadratic Taylor approximation to evaluate the exponential.

5. Compute the volume above the unit square $0 \leq x, y \leq 1$ and underneath a canopy whose height at the point (x, y) is given by the function xe^{xy}.

6. Sketch the region of integration and evaluate the double integral.

$$\int_0^5 \int_0^{10-2x} \left(1 + \frac{xy}{50}\right)\, dy\, dx$$

7. (a) For what value of C is Cye^{-x} a probability density on the infinite strip $0 \leq y \leq 1, 0 \leq x < \infty$?

(b) What is the mean of the Y variable for this probability density?

(c) (4 points extra credit, if you have time) What is the probability that $Y \leq X$ for a pair of values (X, Y) drawn from this density? (Show your work on the facing page)

8. Let $f(x, y) = \ln(x + \sqrt{y})$.

   (a) Compute $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$.

   (b) Evaluate these at $x = 1/2$ and $y = 9/4$.

   (c) Use the Increment Theorem with $\Delta x = 1/10$ and $\Delta y = -1/4$ to estimate $\ln(0.6 + \sqrt{2})$.

9. Use the chain rule to evaluate $\dfrac{\partial f}{\partial s}$ at the point $(s,t) = (4,7)$ if:

$$\begin{aligned} x &= s^2 - 2t + 1 \\ y &= \ln(2s - t) \\ f(x,y) &= \sqrt{x + e^y} \end{aligned}$$

10. Suppose the price P that that the average buyer is willing to pay for a new sports car is proportional to $x^3/\sqrt{y}$ where x is the maximum speed the car can attain in MPH and y is the average number of fatalities per million hours driven. For each part (a)–(c), please state what quantity needs to be computed to answer the given question, then compute it.

    (a) What is the amount that the sales price will increase per 1MPH increase in the maximum speed? This will depend on x and y and perhaps other variables or constants; you should start by writing down an equation satisfied by P, x and y.

    (b) Compute the amount that the sales price will increase for each increase of 1 fatality per million hours driven in the consumer safety data.

    (c) Suppose that the actual maximum speed is 180 MPH and that the number of fatalities per million was estimated at 6.0. If new data causes this estimate to be revised upward to 6.4, roughly how much does the maximum speed need to increase in order for the manufacturer to be able to charge the same price as before?

TABLE 8.1 Basic integration formulas

1. $\int k\,dx = kx + C$ (any number k)

2. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \qquad (n \neq -1)$

3. $\int \frac{dx}{x} = \ln|x| + C$

4. $\int e^x\,dx = e^x + C$

5. $\int a^x\,dx = \frac{a^x}{\ln a} + C \qquad (a > 0,\ a \neq 1)$

6. $\int \sin x\,dx = -\cos x + C$

7. $\int \cos x\,dx = \sin x + C$

8. $\int \sec^2 x\,dx = \tan x + C$

9. $\int \csc^2 x\,dx = -\cot x + C$

10. $\int \sec x \tan x\,dx = \sec x + C$

11. $\int \csc x \cot x\,dx = -\csc x + C$

12. $\int \tan x\,dx = \ln|\sec x| + C$

13. $\int \cot x\,dx = \ln|\sin x| + C$

14. $\int \sec x\,dx = \ln|\sec x + \tan x| + C$

15. $\int \csc x\,dx = -\ln|\csc x + \cot x| + C$

16. $\int \sinh x\,dx = \cosh x + C$

17. $\int \cosh x\,dx = \sinh x + C$

18. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$

19. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

20. $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + C$

21. $\int \frac{dx}{\sqrt{a^2 + x^2}} = \sinh^{-1}\left(\frac{x}{a}\right) + C \qquad (a >$

22. $\int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1}\left(\frac{x}{a}\right) + C \qquad (x >$