Name: ____

Midterm Exam II for Math 110, Spring 2015

March 26, 2015

Problem	Points	Score
1	12	
2	12	
3	12	
4	12	
5	12	
6	12	
7	12	
8	12	
9	12	
10	12	
Total	120	

- You have ninety minutes for this exam.
- Please show **ALL** your work on this exam paper. Partial credit will be awarded where appropriate.
- **CLEARLY** indicate final answers. Use words (doesn't have to be that many words) to connect mathematical formulas and equations.
- **NO** books, notes, laptops, cell phones, calculators, or any other electronic devices may be used during the exam. One 8.5×11 cheatsheet, handwritten is allowed; it may be double-sided.
- No form of cheating will be tolerated. You are expected to uphold the Code of Academic Integrity.

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this midterm examination.

Signature: ________________________________

Date: ________________________________

1. (a) Write this integral as a limit or a sum of limits.

$$\int_{-\infty}^{\infty} \frac{dx}{\sqrt{1+x^2+x^4}}$$

(b) State whether the integral converges and give a justification of your answer.

2. (a) For what value of C is $f(x) = Cxe^{-x/2}$ a probability density on the half line $[0, \infty)$? Please show how you evaluate any integrals.

(b) What is the mean of this density?

3. (a) Compute the quadratic Taylor polynomial $P_2(x)$ for the function $f(x) = \frac{1}{\sqrt{x}}$ about the point $x = 9$.

(b) What approximation does this give for $\frac{1}{\sqrt{10}}$? Write the answer in the box, simplifying fractions when possible:

$P_2(10) = \boxed{}$

(c) State what Taylor's theorem with remainder says about the remainder $R_2 = \frac{1}{\sqrt{10}} - P_2(10)$. You do not need to compute anything or find bounds, just state the conclusion of the theorem applied to this case.

4. Write an initial value problem for this scenario. You do not have to solve the differential equation but you must give the interpretation of all variables and constants, their units, and indicate which is the dependent and the independent variable.

   > Toxins accumulate at a toxic waste site at an average rate of 1.9 kg per hour. Natural processes are able to break down the toxins at a rate proportional to the amount of toxins present, the time constant being 0.10 yr^{-1}. The site opened in 1995 when an area containing 4,800 kg of toxins was dedicated to toxic waste storage.

5. (a) Does this series converge? Justify your answer.

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln(n)}$$

(b) Does this series converge? Justify your answer.

$$\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln(n)}}$$

6. Roberts' law for executive compensation says that an executive of a firm of size S earns an amount proportional to $S^{1/3}$. In order to study this further, an economist compiles a list of 1,000 firms, one of each size from 1 to 1,000.

   (a) Write an expression in Σ-notation for the total executive compensation among the firms in the study. State which variables in your sum are free and which are bound.

   (b) Give an approximate value for this sum by approximating it by an integral; free variables should remain unevaluated.

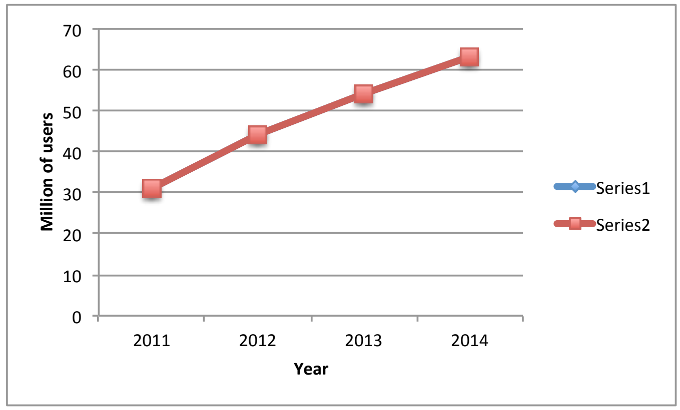
   (c) Say whether the actual sum is greater or less than your integral approximation.

7. Write an initial value problem for this integral equation.

$$y(t) = 3t + \int_2^t \frac{1}{s + y(s)}\, ds\,.$$

8. The number of iPhone users in the US in the years 2011–2014 is given by the following chart[1].

2011	31 million
2012	44 million
2013	54 million
2014	63 million



(a) (8 points) Circle which of the following differential equations you think best models this information, if t is time in years and y is users in millions? This part will be graded on an all or nothing basis unless you choose to provide a justification.

(i) $\dfrac{dy}{dt} = y + 10.3$

(ii) $\dfrac{dy}{dt} = y + 10.3t$

(iii) $\dfrac{dy}{dt} = y + 10.3(t - 2011)$

(iv) $\dfrac{dy}{dt} = 30 + 10.3(t - 2011)$

(v) $\dfrac{dy}{dt} = y - t$

(vi) $\dfrac{dy}{dt} = 14 - \sqrt{y - 28}$

(b) (4 points) For your choice of differential equation, will $y(t)$ grow forever or approach a limiting value? Justify your answer by saying where the slope field slopes up versus down.

[1]Data is taken from http://www.statista.com/statistics/232790/forecast-of-apple-users-in-the-us

9. Use Euler iteration with a step size of $1/3$ to approximate $y(2)$ where $y(t)$ is the solution to the initial value problem

$$y' = 3x - y\,; \quad y(1) = 2\,.$$

10. (a) Compute the MacLaurin series for $e^{-x^2/2}$ and write the result in Σ-notation.

(b) For what x does this series converge?

(c) What is the quartic Taylor polynomial $P_4(x)$?

TABLE 8.1 Basic integration formulas

1. $\int k\,dx = kx + C$ (any number k)
2. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$ $(n \neq -1)$
3. $\int \frac{dx}{x} = \ln|x| + C$
4. $\int e^x\,dx = e^x + C$
5. $\int a^x\,dx = \frac{a^x}{\ln a} + C$ $(a > 0,\ a \neq 1)$
6. $\int \sin x\,dx = -\cos x + C$
7. $\int \cos x\,dx = \sin x + C$
8. $\int \sec^2 x\,dx = \tan x + C$
9. $\int \csc^2 x\,dx = -\cot x + C$
10. $\int \sec x \tan x\,dx = \sec x + C$
11. $\int \csc x \cot x\,dx = -\csc x + C$
12. $\int \tan x\,dx = \ln|\sec x| + C$
13. $\int \cot x\,dx = \ln|\sin x| + C$
14. $\int \sec x\,dx = \ln|\sec x + \tan x| + C$
15. $\int \csc x\,dx = -\ln|\csc x + \cot x| + C$
16. $\int \sinh x\,dx = \cosh x + C$
17. $\int \cosh x\,dx = \sinh x + C$
18. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$
19. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$
20. $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + C$
21. $\int \frac{dx}{\sqrt{a^2 + x^2}} = \sinh^{-1}\left(\frac{x}{a}\right) + C$ $(a >$
22. $\int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1}\left(\frac{x}{a}\right) + C$ $(x >$

Logarithm Cheat Sheet

These values are accurate to within 1%:

$$\begin{aligned} e &\approx 2.7 \\ \ln(2) &\approx 0.7 \\ \ln(10) &\approx 2.3 \\ \log_{10}(2) &\approx 0.3 \\ \log_{10}(3) &\approx 0.48 \end{aligned}$$

Some other useful quantities to with 1%:

$$\begin{aligned} \pi &\approx \frac{22}{7} \\ \sqrt{10} &\approx \pi \\ \sqrt{2} &\approx 1.4 \\ \sqrt{1/2} &\approx 0.7 \end{aligned}$$

(ok so technically $\sqrt{2}$ is about 1.005% greater than 1.4 and 0.7 is about 1.005% less than $\sqrt{1/2}$)