Name: ______________________________ Section (circle one): 001 002

Midterm Exam II for Math 110, Fall 2014

November 4, 2014

Problem	Points	Score
1	12	
2	12	
3	12	
4	12	
5	12	
6	12	
7	12	
8	12	
9	12	
10	12	
Total	120	

- You have ninety minutes for this exam.
- Please show **ALL** your work on this exam paper. Partial credit will be awarded where appropriate.
- **CLEARLY** indicate final answers. Use words (doesn't have to be that many words) to connect mathematical formulas and equations.
- **NO** books, notes, laptops, cell phones, calculators, or any other electronic devices may be used during the exam. One 8.5×11 cheatsheet, handwritten is allowed; it may be double-sided.
- No form of cheating will be tolerated. You are expected to uphold the Code of Academic Integrity.

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this midterm examination.

Signature: ______________________

Date: ______________________

1. Write the following indefinite integrals as limits. Do not attempt to evaluate them.

   (a)
   $$\int_0^1 \ln x \, dx \, .$$

   (b)
   $$\int_{-\infty}^{\infty} e^{1/(1+x^2)} \, dx$$

2. (a) For what value of C is Cx^{-5} a probability distribution on $[1, \infty)$?

(b) What is the mean of this distribution?

3. (a) Compute the quadratic Taylor polynomial $P_2(x)$ for the function $f(x) = x^{1/3}$ about the point $x = 1$.

(b) Use this to estimate $\sqrt[3]{1.3}$ and write the answer in the box:

$P_2(1.3) =$ [ ]

(c) State what Taylor's theorem with remainder says about the remainder $R_2 = 1.3^{1/3} - P_2(1.3)$. You do not need to compute anything or find bounds.

4. Compute the quadratic Taylor polynomial $P_2(x)$ for the function

$$f(x) = \int_3^x \ln(1 + t^2)\, dt$$

about the point $x = 3$. Do not evaluate fractions, radicals, logarithems and so forth as decimals: you should simplify if possible, leaving expressions such as $\sqrt{2}/2$, $\ln 5$, etc.

5. For which values of x does the series $\sum_{n=1}^{\infty} 3^{n/2} x^n$ converge?

6. (a) Compute the first four nonzero terms of the Taylor polynomial for $e^{x^2/2}$ around $x = 0$. [Hint: substituion is easier than computing the derivatives directly.]

(b) Write the Taylor series as a sum in Sigma notation. Examples of such series are $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-1)^n}{n}$ and $\sum_{n=0}^{\infty} \frac{x^n}{n!}$.

7. Write a differential equation or initial value problem for this scenario. Be sure to give interpretations and units for every variable and constant.

    When a nuclear reactor becomes hotter than its "critical temperature", its temperature increases at a rate which remains proportional to the fourth power of the amount by which the critical temperature is exceeded.

8. (a) Find the general solution of the differential equation

$$\frac{dy}{dt} = k \cdot (125 - y)\,.$$

(b) If $y(1) = 50$ and $y(3) = 122$ then what is $y(0)$?

9.

$$\frac{dy}{dx} = y - x, \quad y(0) = 1\,.$$

Approximate $y(1)$ by using Euler iteration with step size $1/3$. Please leave everything in exact form (fractions or radicals rather than decimals).

10. (a) Choose which differential equation is depicted in this slope field. You don't need to write anything, just circle one of the choices.

(i) $y' = y - (x-2)^2$

(ii) $y' = 4 - y - x$

(iii) $y' = \dfrac{x}{1+y}$

(iv) $y' = (x-2)^2 - y$

(v) $y' = \dfrac{4}{1+y}$

(b) If, in addition, you are given that $y(0) = 3$ then which of the choices best approximates $y(3)$? Justify your answer by sketching the solution onto the given slopefield.

(i) 3.6

(ii) 3.0

(iii) 2.4

(iv) 1.8

(v) 1.2

(vi) 0.6

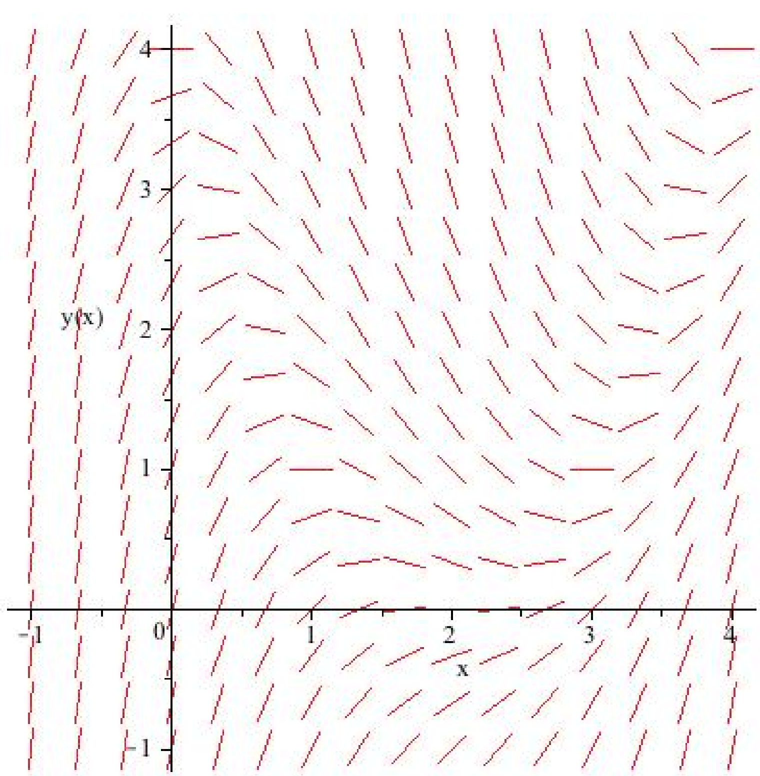

TABLE 8.1 Basic integration formulas

1. $\int k \, dx = kx + C$ (any number k)

2. $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$ $(n \neq -1)$

3. $\int \frac{dx}{x} = \ln |x| + C$

4. $\int e^x \, dx = e^x + C$

5. $\int a^x \, dx = \frac{a^x}{\ln a} + C$ $(a > 0, \ a \neq 1)$

6. $\int \sin x \, dx = -\cos x + C$

7. $\int \cos x \, dx = \sin x + C$

8. $\int \sec^2 x \, dx = \tan x + C$

9. $\int \csc^2 x \, dx = -\cot x + C$

10. $\int \sec x \tan x \, dx = \sec x + C$

11. $\int \csc x \cot x \, dx = -\csc x + C$

12. $\int \tan x \, dx = \ln |\sec x| + C$

13. $\int \cot x \, dx = \ln |\sin x| + C$

14. $\int \sec x \, dx = \ln |\sec x + \tan x| + C$

15. $\int \csc x \, dx = -\ln |\csc x + \cot x| + C$

16. $\int \sinh x \, dx = \cosh x + C$

17. $\int \cosh x \, dx = \sinh x + C$

18. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$

19. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$

20. $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1}\left|\frac{x}{a}\right| + C$

21. $\int \frac{dx}{\sqrt{a^2 + x^2}} = \sinh^{-1}\left(\frac{x}{a}\right) + C$ $(a >$

22. $\int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1}\left(\frac{x}{a}\right) + C$ $(x >$