Name: ____

Midterm Exam I for Math 110, Spring 2015

February 19, 2015

Problem	Points	Score
1	12	
2	12	
3	12	
4	12	
5	12	
6	12	
7	12	
8	12	
9	12	
10	12	
Total	120	

- You have ninety minutes for this exam.
- Please show **ALL** your work on this exam paper. Partial credit will be awarded where appropriate.
- **CLEARLY** indicate final answers. Use words (doesn't have to be that many words) to connect mathematical formulas and equations.
- **NO** books, notes, laptops, cell phones, calculators, or any other electronic devices may be used during the exam. One 8.5×11 cheatsheet, handwritten is allowed; it may be double-sided.
- No form of cheating will be tolerated. You are expected to uphold the Code of Academic Integrity.

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this midterm examination.

Signature: __________________________________

Date: __________________________________

1. Graph the function $\dfrac{1}{x^2 - 9}$. Please choose scales on the x- and y-axes that are neither too small nor too large to show the shape, and point out any maxima, minima and asymptotes, as well as exact coordinates of a few points on the graph.

2. What is the relation between x and y if $\log_{10} x = 1 - \log_{10} y$?

3. The number of calories consumed per day by a mammal is roughly proportional to the 2/3 power of its volume.

   (a) Write an equation expressing this. Separately, state the interpretation of each variable or constant and its units.

   (b) By what factor does the consumption increase if the animal quadruples in volume? Please leave this as an exact expression, not a decimal approximation.

   (c) Give an approximate decimal value by using your log cheatsheet. It comes out easiest if you use base-ten logs.

4. Find a function $g(x) = cx^p$ such that $f(x) \sim g(x)$ as $x \to \infty$, where

$$f(x) = \sqrt{25x^3 + x^2 + e^{-x}}\,.$$

You do not have to prove your answer.

5. True or false?

(a) $\ln x \ll x^{1/8}$ as $x \to \infty$

(b) $x^{1/2} = o(x^{1/3})$ as $x \to 0^+$

(c) $x^{1/2} = o(x^{1/3})$ as $x \to \infty$

(d) $\sqrt{1 + x^4} \sim x^2$ as $x \to \infty$

6. In each case, write the sum (call is S) in Σ notation, then evaluate it (leave in exact form, do not use decimal approximations).

   (a) $M + (M - 13) + (M - 26) + \cdots + (M - 130)$

   (b) $\frac{3}{5} - \frac{3}{10} + \frac{3}{20} - \frac{3}{40} + \cdots + \cdots$ going on forever

7. Let $A = \displaystyle\int_1^6 \frac{1}{x}\,dx$. Write, in Σ notation, a right-Riemann sum for this integral that has five terms. Evaluate this as a fraction. Draw the Riemann sum and the function on the same graph and use this drawing to say whether the Riemann sum is greater or less than A.

8. Compute the indefinite integral

$$\int \frac{\cos x}{\sqrt{1 + \sin x}}\, dx\,.$$

9. The predicted yield of an oil rig in barrels per day, when it has been x days in operation, is $f(x) = x^2 e^{-x/100}$.

   (a) Compute the exact value of $\int_{100}^{400} f(x)\, dx$.

   (b) State what physical quantity this represents.

   (c) To the nearest million (a very crude estimate, no pun intended), how many barrels is this?

10. Joe's credit card charges 2.0% interest per month. There is a minimum payment of \$100 per month to avoid a further finance charge. Suppose Joe charges \$300 per month and never pays more than the minimum (see footnote for exact timing[1]).

    (a) How much does he owe after three months of this?

    (b) Write an expression in Sigma notation for what Joe owes after five years.

    (c) Evaluate this expression analytically – that means you will have expressions with powers in them.

    (d) Find a numerical approximation to this expression by using ln and its linearization near 1.

[1] Assume that the charges and the payment appear on the last day of the month, and that interest on this part of the balance is computed at the end of the next month.

TABLE 8.1 Basic integration formulas

1. $\int k\,dx = kx + C$ (any number k)

2. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$ $(n \neq -1)$

3. $\int \frac{dx}{x} = \ln|x| + C$

4. $\int e^x\,dx = e^x + C$

5. $\int a^x\,dx = \frac{a^x}{\ln a} + C$ $(a > 0,\ a \neq 1)$

6. $\int \sin x\,dx = -\cos x + C$

7. $\int \cos x\,dx = \sin x + C$

8. $\int \sec^2 x\,dx = \tan x + C$

9. $\int \csc^2 x\,dx = -\cot x + C$

10. $\int \sec x \tan x\,dx = \sec x + C$

11. $\int \csc x \cot x\,dx = -\csc x + C$

12. $\int \tan x\,dx = \ln|\sec x| + C$

13. $\int \cot x\,dx = \ln|\sin x| + C$

14. $\int \sec x\,dx = \ln|\sec x + \tan x| + C$

15. $\int \csc x\,dx = -\ln|\csc x + \cot x| + C$

16. $\int \sinh x\,dx = \cosh x + C$

17. $\int \cosh x\,dx = \sinh x + C$

18. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$

19. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

20. $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + C$

21. $\int \frac{dx}{\sqrt{a^2 + x^2}} = \sinh^{-1}\left(\frac{x}{a}\right) + C$ $(a >$

22. $\int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1}\left(\frac{x}{a}\right) + C$ $(x >$

Logarithm Cheat Sheet

These values are accurate to within 1%:

$$\begin{aligned} e &\approx 2.7 \\ \ln(2) &\approx 0.7 \\ \ln(10) &\approx 2.3 \\ \log_{10}(2) &\approx 0.3 \\ \log_{10}(3) &\approx 0.48 \end{aligned}$$

Some other useful quantities to with 1%:

$$\begin{aligned} \pi &\approx \frac{22}{7} \\ \sqrt{10} &\approx \pi \\ \sqrt{2} &\approx 1.4 \\ \sqrt{1/2} &\approx 0.7 \end{aligned}$$

(ok so technically $\sqrt{2}$ is about 1.005% greater than 1.4 and 0.7 is about 1.005% less than $\sqrt{1/2}$)