Name: _____ Section (circle one): 001 002

Midterm Exam I for Math 110, Fall 2014

September 30, 2014

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
Total	120	

- You have ninety minutes for this exam.
- Please show **ALL** your work on this exam paper. Partial credit will be awarded where appropriate.
- **CLEARLY** indicate final answers. Use words (doesn't have to be that many words) to connect mathematical formulas and equations.
- **NO** books, notes, laptops, cell phones, calculators, or any other electronic devices may be used during the exam. One 8.5×11 cheatsheet, handwritten is allowed; it may be double-sided.
- No form of cheating will be tolerated. You are expected to uphold the Code of Academic Integrity.

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this midterm examination.

Signature: _____

Date: _____

1. Graph the function xe^{-x} from $-\infty$ to ∞. Be sure to indicate any maxima, minima or asymptotes.

2. (a) Write an equation stating that the surface area of a microchip is proportional to the $2/3$ power of its volume. Be sure to state the units and the meaning of each variable and constant.

(b) The new microchip has one tenth the volume of the old microchip. What is the ratio of the surface area of the new chip to that of the old chip.

Please leave the answer in the form of a power; a decimal value is not needed.

3. Compute the inverse function for $f(x) = 5 + e^{2x}$ and give the domain and range of the inverse function.

4. (a) Use the linearization of $\log_{10} x$ near $a = 1$ to estimate $\log_{10} 1.046$.

(b) Use this to estimate $\log_{10}\left(1.046^{65}\right)$.

(c) Use this to estimate 1.046^{65}.

5. How many digits does the number e^{47} have? Circle the correct answer. If you answer with no justification you get all or nothing. If you provide justification you may get partial credit, but to get full credit it has to be correct.

(i) 19

(ii) 20

(iii) 21

(iv) 46

(v) 47

(vi) 48

6. Find a function g of the form $g(x) = cx^p$ such that $f(x) \sim g(x)$ as $x \to \infty$ if f is the function defined by

$$f(x) = \frac{3x^2 + 1}{6x^5 + 2x^4 + 7x^3 - 9x + 9}\,.$$

7. Write the infinite sum $\frac{1}{14} - \frac{1}{28} + \frac{1}{56} - \frac{1}{112} + \cdots$ in Sigma notation and evaluate it.

   Please leave the result as a fraction.

8. On January 1, in each year from 2001 to 2014, my grandma gave me a Certificate of Deposit for \$1,000 that grows at the rate of 5% per year. On January 1, 2015 (she did not give me one in 2015) what is the total value of these CD's?

   Please leave the answer as an exact expression in powers, fractions, etc.

9. Approximate $\int_0^{\pi/2} \sin^4 x \, dx$ in the following three ways. In parts (a)–(c) you should report the exact value (that is, leave in terms of π, fractions, trig functions, etc.).

   (a) A lower Riemann sum with three rectangles

   (b) An upper Rieman sum with three rectangles

(c) A trapezoidal approximation with three trapezoids (triangles count as trapezoids)

(d) Circle which of these answers you believe to be closest to the numerical value.

- 0.2
- 0.4
- 0.6
- 0.8
- 1.0
- 1.2

10. Compute these integrals.

(a) $\displaystyle\int \cos x\sqrt{1+\sin x}\,dx$

(b) $\displaystyle\int_0^{\pi/4} e^{\tan\theta}\,\sec^2\theta\,d\theta$

11. Compute the indefinite integral $\int x \sec^2 x dx$.

12. The arrival rate of a flash mob in people per minute is $100\,t\,e^{-t}$ where t is the time in minutes since the message went out. After ten minutes, the mob gives a final shout and disperses. Approximately how many people are present for the final shout?

Logarithm Cheat Sheet

These values are accurate to within 1%:

$$\begin{aligned} e &\approx 2.7 \\ \ln(2) &\approx 0.7 \\ \ln(10) &\approx 2.3 \\ \log_{10}(2) &\approx 0.3 \end{aligned}$$

Here are some values of base-ten logs to three decimal places:

$$\begin{aligned} \log_{10}(2) &\approx 0.301 \\ \log_{10}(3) &\approx 0.477 \\ \log_{10}(4) &\approx 0.602 \\ \log_{10}(5) &\approx 0.699 \\ \log_{10}(6) &\approx 0.778 \\ \log_{10}(7) &\approx 0.845 \\ \log_{10}(8) &\approx 0.903 \\ \log_{10}(9) &\approx 0.954 \end{aligned}$$

Some other useful quantities to with 1% or so:

$$\begin{aligned} \pi &\approx \frac{22}{7} \\ \sqrt{10} &\approx \pi \\ \sqrt{2} &\approx 1.4 \\ \sqrt{1/2} &\approx 0.7 \\ \sqrt{3} &\approx 1.732 \\ \sqrt{5} &\approx 2.236 \end{aligned}$$

TABLE 8.1 Basic integration formulas

1. $\int k\,dx = kx + C$ (any number k)

2. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \qquad (n \neq -1)$

3. $\int \frac{dx}{x} = \ln|x| + C$

4. $\int e^x\,dx = e^x + C$

5. $\int a^x\,dx = \frac{a^x}{\ln a} + C \qquad (a > 0,\ a \neq 1)$

6. $\int \sin x\,dx = -\cos x + C$

7. $\int \cos x\,dx = \sin x + C$

8. $\int \sec^2 x\,dx = \tan x + C$

9. $\int \csc^2 x\,dx = -\cot x + C$

10. $\int \sec x \tan x\,dx = \sec x + C$

11. $\int \csc x \cot x\,dx = -\csc x + C$

12. $\int \tan x\,dx = \ln|\sec x| + C$

13. $\int \cot x\,dx = \ln|\sin x| + C$

14. $\int \sec x\,dx = \ln|\sec x + \tan x| +$

15. $\int \csc x\,dx = -\ln|\csc x + \cot x| +$

16. $\int \sinh x\,dx = \cosh x + C$

17. $\int \cosh x\,dx = \sinh x + C$

18. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$

19. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

20. $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + C$

21. $\int \frac{dx}{\sqrt{a^2 + x^2}} = \sinh^{-1}\left(\frac{x}{a}\right) + C$

22. $\int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1}\left(\frac{x}{a}\right) + C$