MATH 104 – Sample Final Exam 2

1. A scientist collects data that relate two variables, x and y. Instead of plotting y as a function of x, she plots $\log_2 y$ as a function of x, and gets a line whose slope is 3 and whose intercept on the vertical axis is 4. What equation describes y as a function of x?

   (a) $y = 3x + 4$ (b) $y = 3x + 16$ (c) $y = 16 \cdot 8^x$
   (d) $y = 16e^{3x}$ (e) $y = 4x^3$ (f) $y = 16x^3$

2. A solid has a circular base of radius 1. Parallel cross-sections perpendicular to the base are squares. The volume of the solid is:

   (a) $1/3$ (b) $\pi/2$ (c) $4\pi/3$ (d) $4/3$ (e) $16/3$ (f) $8\pi/3$

3. $\lim_{n\to\infty} \sum_{i=1}^{n-1} \frac{1}{n} 4(i/n)^2 =$

   (a) 4 (b) e (c) π (d) $4/3$ (e) $8/3$ (f) ∞

4. $\int_1^{\infty} \frac{\ln x}{x^3}\, dx =$

   (a) $1/4$ (b) $1/3$ (c) 1 (d) $\ln 2$ (e) $\ln 3$ (f) divergent

5. $\int_3^4 \frac{4x - 6}{x^2 - 3x + 2}\, dx =$

   (a) $\ln(4/3)$ (b) $2 + \arctan(3)$ (c) $\ln(9)$
   (d) $\ln(12/5)$ (e) $\pi/3 - \arctan(1/4)$ (f) $\pi/4 - \ln(3)$

6. Consider the two infinite series: (I) $\sum_{n=1}^{\infty} \frac{(-1)^n (n+1) 3^n}{2^{2n+1}}$, $\sum_{n=1}^{\infty} \frac{(-1)^n \ln(n)}{\sqrt{n}}$

   (a) Both converge absolutely.
   (b) Both converge conditionally.
   (c) Both diverge.
   (d) I converges absolutely and II diverges.
   (e) I converges absolutely and II converges conditionally.
   (f) I converges conditionally and II diverges.

7. Evaluate $\lim_{x\to\infty} \frac{\int_1^x \sqrt{9 + e^{-2t}}\, dt}{x}$.

   (a) 0 (b) 1 (c) 3 (d) 9 (e) $3e$ (f) does not exist

8. The first few terms of the Maclaurin series for $\int_0^x \sqrt{1+t^3}\,dt$ are:

(a) $1+\frac{x^3}{2}-\frac{x^6}{8}+\cdots$ (b) $x+\frac{x^4}{8}-\frac{x^7}{56}+\cdots$
(c) $x+\frac{x^2}{2}-\frac{x^3}{8}+\cdots$ (d) $1+\frac{x}{2}-\frac{x^2}{8}+\cdots$
(e) $x+\frac{x^2}{4}-\frac{x^3}{24}+\cdots$ (f) $1+\frac{x}{4}-\frac{x^3}{8}+\cdots$

9. The interval of convergence of the series $\sum_{n=1}^{\infty}\frac{2^n(x-3)^n}{\sqrt{n}}$ is:

(a) $(1,5)$ (b) $[-7/2,7/2]$ (c) $[-1/2,1/2)$
(d) $(5/2,7/2)$ (e) $(2,4]$ (f) $[5/2,7/2)$

10. To the nearest 0.0001, the value of $\int_{-0.1}^{0.1}\frac{1-e^{-x^2}}{x^2}\,dx$ is

(a) 0.1997 (b) 0.1998 (c) 0.1999 (d) 0.2000 (e) 0.2001 (f) 0.2002

11. If the function $y=f(x)$ satisfies the differential equation $\frac{dy}{dx}+x^2y=x^2$ and if $f(0)=5$, then $f(1)=$

(a) e^3-1 (b) $\frac{3e}{1+e}$ (c) $5e^{-1/2}$
(d) $\sqrt{2}$ (e) $5+e^{-1}$ (f) $1+4e^{-1/3}$

12. A tank contains 1000 liters of brine with 50 kilograms of dissolved salt. Pure water enters the tank at the rate of 25 liters per minute. The solution is kept thoroughly mixed and drains at an equal rate. How many kilograms of salt remain after 10 minutes?

(a) 0 (b) 50 (c) $50e^{0.25}$
(d) $50e^{-0.25}$ (e) $25e^{-0.1}$ (f) $50\ln 2$

13. $\int_0^{\infty}\frac{x}{1+x^4}\,dx=$

(a) 0 (b) π (c) $\pi/2$
(d) $\pi/4$ (e) $\pi/8$ (f) diverges

14. A bacteria colony starts with 200 bacteria and in one hour contains 400 bacteria. How many hours (from the initial time) does it take to reach 2000 bacteria?

(a) $\ln 400$ (b) $\ln 10$ (c) $\ln 200$
(d) $\ln 2000/\ln 400$ (e) $\ln 10/\ln 2$ (f) $\ln 2000/\ln 200$

15. Let $f(x) = \dfrac{e^x + e^{-x}}{2}$. Which of the following is true?

(I) $f(x) = |f'(x)|$
(II) The graph of $y = f(x)$ is concave up for all x.
(III) $f(x)$ is increasing for $x > 0$.

(a) I only (b) I and II (c) I and III
(d) II and III (e) I, II and III (f) II only

16. A local politician has noticed that the amount of money contributed to his campaign fund at a political rally is related to the length of his speech. If his speech is 10 minutes long, 200 people will attend the rally and give an average of \$12 each. For every minute over 10 that his speech lasts, the individual giving goes up by \$0.50 per minute, but also for each minute longer the speech lasts, the number of people attending the rally drops by 10. How long should his speeches be if he wishes to maximize the total contributions?

(a) 8 minutes (b) 9 minutes (c) 10 minutes
(d) 11 minutes (e) 12 minutes (f) 13 minutes

17. Evaluate $\displaystyle\int_0^\infty xe^{-4x}\,dx$.

(a) divergent (b) 1/4 (c) 1/3 (d) 4 (e) 1/8 (f) 1/16

18. The value of the partial derivative $\dfrac{\partial^2 f}{\partial x \partial y}$ of the function $f(x, y) = \ln(x + \ln y)$ at the point (2,1) is:

(a) $-1/4$ (b) 1/4 (c) $-1/2$ (d) 1/2 (e) $-3/4$ (f) 3/4

19. Find all the points where the function $f(x, y) = x^4 + y^4 - 4xy$ has a local minimum.

(a) (0,0) (b) (0,0) and (1,1) (c) $(-1,-1)$ and (0,0)
(d) $(-1,-1)$ and (1,1) (e) $(-1,-1)$, (0,0) and (1,1) (f) f has no local minima

20. Let $f(x) = e^{2x^3} + x\cos(3x^2)$. What is $f^{(9)}(0)$ (that is, the ninth derivative of f evaluated at 0)? Hint: Don't even *think* of differentiating the function nine times!

(a) $9!\frac{31}{8}$ (b) $9!\frac{85}{24}$ (c) $9!\frac{5}{24}$ (d) $9!\frac{113}{24}$ (e) $9!\frac{11}{8}$ (f) $9!\frac{49}{24}$