MATH 104 – Sample Final Exam 1

1. A scientist collects data that relate two variables, x and y. Instead of plotting y as a function of x, she plots $\log_{10} y$ as a function of $\log_{10} x$, and gets a line whose slope is 3 and whose intercept on the vertical axis is 2. What equation describes y as a function of x?

   (a) $y = 3x + 2$ (b) $y = 3x + 100$ (c) $y = 100e^{3x}$ (d) $y = 2x^3$ (e) $y = 100x^3$

2. What is the volume of the solid generated by rotating the region bounded by the x-axis, the curve $y = \ln x$ and the line $x = e$ around the y-axis?

   (a) $\pi e - 2\pi$ (b) $\dfrac{\pi(e^2 - 1)}{2}$ (c) $\ln(\pi) - \frac{1}{2}$ (d) $\dfrac{\pi(e^2 + 1)}{2}$ (e) $\ln 2 - \ln \pi$

3. $\displaystyle\lim_{n\to\infty} \left(1 + \frac{2}{n}\right)^{3n} =$

   (a) 1 (b) e (c) e^5 (d) e^6 (e) ∞

4. $\displaystyle\int_0^{\pi} \cos^4 x \, dx =$

   (a) 2 (b) π (c) $\pi - \frac{1}{2}$ (d) $\sqrt{2}\pi$ (e) $3\pi/8$

5. $\displaystyle\int_0^1 x^3\sqrt{1 - x^2}\, dx =$

   (a) 1/4 (b) 2/15 (c) $\sqrt{3/2}$ (d) $\pi/6$ (e) 1

6. Consider the two infinite series: (I) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n \sin(3n)}{n^2}$, $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2 + 2}$

   (a) Both converge absolutely (b) Both converge conditionally
   (c) Both diverge (d) I converges absolutely and II diverges
   (e) I converges absolutely and II converges conditionally

7. Evaluate $\displaystyle\lim_{x\to 0} \frac{e^{x^2} - 1 - x^2}{x \sin x - x^2}$. (*Hint*: Use Taylor series.)

   (a) 0 (b) 3 (c) $-1/3$ (d) -3 (e) does not exist

8. Which of the following is the Maclaurin series for $\int_0^x \frac{\sin t - t}{t^3}\, dt$?

(a) $-\frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$
(b) $-\frac{x^3}{3\cdot 5!} + \frac{x^5}{5\cdot 7!} - \frac{x^7}{7\cdot 9!} + \cdots$
(c) $-\frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$
(d) $-\frac{x}{3!} + \frac{x^3}{3\cdot 5!} - \frac{x^5}{5\cdot 7!} + \cdots$
(e) $-\frac{x^2}{2\cdot 3!} + \frac{x^4}{4\cdot 5!} - \frac{x^5}{5\cdot 6!} + \cdots$

9. The interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(x+2)^n}{n(-3)^n}$ is:

(a) $-5 < x \leq 1$ (b) $-5 \leq x < 1$ (c) $-3 \leq x \leq 3$ (d) $-1 < x \leq 5$ (e) $-1 \leq x \leq 5$

10. Bill, Gwen, Sue and Zach use the approximation $e^x \approx 1 + x + \frac{x^2}{2} + \frac{x^3}{6}$ with $x = 0.2$ to compute $e^{0.2}$. They make the following assertions about the error E:

Bill: $|E| < 0.0004$, Gwen: $|E| < 0.0003$
Sue: $|E| < 0.0002$, Zach: $|E| < 0.00001$

Which of them are correct? (Note: $1 < e < 3$)

(a) only Bill (b) only Bill and Gwen (c) only Bill, Gwen and Sue
(d) all of them (e) none of them

11. Let $y(x)$ be the solution of $\frac{dy}{dx} - 2y = 6$ such that $y(0) = 1$. Then $y(1)$ is:

(a) $(3 - e^2)/2$ (b) $e^{-2} - 3$ (c) $e^2 + 3$ (d) $4e^2 - 3$ (e) $-2e^2 - 3$

12. A thermometer is taken from a room where the temperature is 20° C to the outdoors where the temperature is 5° C. After one minute, the thermometer read 12° C. After how many minutes (after being taken outdoors) will the thermometer read 6° C?

(a) $\ln 15$ (b) $\ln 7/\ln 15$ (c) $\ln 15/\ln 7$
(d) $\ln 15/(\ln 15 - \ln 7)$ (e) $\ln 7/(\ln 15 - \ln 7)$

13. $\int_0^{\infty} \frac{1}{\sqrt{x}(1+x)}\, dx =$

(a) 0 (b) π (c) $\pi/2$ (d) $e/2$ (e) diverges

14. A couple plans to invest money at a constant rate of c dollars per year for $T = 20$ years. The interest rate of their investment is 5% per year, compounded continuously. What value of c will make the value of their investment equal to \$20,000 at $T = 20$ years?

(a) $20,000/(e-1)$ (b) $1,000/(e-1)$ (c) $1,000 \cdot \ln 2$
(d) $20,000/\ln(20)$ (e) $20,000e^{-1}$

15. Let $f(x) = \dfrac{1}{1-e^x}$. Which of the following is true?

(i) The graph of f has a horizontal asymptote at $y = 1$.
(ii) The graph of f has a vertical asymptote at $x = 1$.
(iii) f is decreasing for $x > 0$.
(iv) f is concave upward for $x < 0$.

(a) (i) and (ii) (b) only (i) (c) (iii) and (iv)
(d) (i) and (iv) (e) (i), (ii), (iii), and (iv)

16. Until recently, hamburgers at a certain sports arena sold for \$4 each. The food concession sold an average of 8,000 hamburgers on a game night. When the price was raised to \$4.25, hamburger sales dropped to an average of 7,000 per night. Meanwhile, the concession's fixed costs are \$2,000 per night and the variable costs are \$1 per hamburger. Assuming the demand curve is linear, find the price of a hamburger that will maximize the average nightly hamburger profit.

(a) \$3.00 (b) \$3.50 (c) \$3.75 (d) \$4.50 (e) \$5.00

17. Evaluate $\displaystyle\int_0^1 x \ln x \, dx$.

(a) $-1/4$ (b) $-1/2$ (c) 0 (d) $1/4$ (e) $1/2$

18. Recall that the distance from a point (x, y, z) in three-dimensional space to the origin is $D = \sqrt{x^2 + y^2 + z^2}$. Compute the *shortest* distance from the origin to a point on the surface $z = \dfrac{1}{xy}$.

(a) 3 (b) $\sqrt{3}$ (c) $\sqrt{2+\sqrt{2}}$ (d) $2 + 2\sqrt{2}$ (e) There is no closest point.

19. The radius of a right circular cylinder is measured with an error of at most 3%, and the height is measured with an error of at most 2%. Using differentials, approximate the maximum percentage error in the volume of the cylinder as calculated from these measurements.

(a) 5% (b) 7% (c) 8% (d) 10% (e) 12%

20. The coefficient of $(x-1)^4$ in the Taylor series centered at $c = 1$ for the function $\ln x$ is

(a) $-1/4!$ (b) $1/3$ (c) 6 (d) $1/4$ (e) $-1/4$