MATH 104 – Final Exam - Fall 2006

1. Find the area between the curves $y = \cos^2 x$ and $y = \sin^2 x$ for x between 0 and the smallest positive value of x for which the two curves intersect.

   (A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) $\frac{3}{2}$ (F) $\frac{3}{4}$

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2. Find the volume of the solid obtained by rotating the region bounded by the curves

$$y = x^{3/2}, \qquad y = 0, \qquad x = 4$$

   about the x-axis.

   (A) 8π (B) $\frac{128\pi}{5}$ (C) 64π (D) 192π (E) 320π (F) 1024π

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3. What is the volume of a solid of revolution generated by rotating around the y-axis the region enclosed by the graph of $y = e^{-x^2}$, the x-axis, and the lines $x = 0$ and $x = 2$?

   (A) πe^{-4} (B) $\pi(1 - e^{-4})$ (C) πe^{-2} (D) $\pi(1 - e^{-2})$ (E) πe^4 (F) $\pi(e^4 - 1)$

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4. Evaluate the integral.

$$\int_1^e x\,(\ln x)^2\,dx$$

   (A) $-\frac{e^2}{4} + e - \frac{1}{4}$ (B) $\frac{e}{6}$ (C) $\frac{e^2}{6}$ (D) $\frac{e^2}{2} - e$ (E) $\frac{e^2}{2} - \frac{e}{2}$ (F) $\frac{e^2}{4} - \frac{1}{4}$

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5. Evaluate the integral.

$$\int_0^1 \frac{dx}{(4 - x^2)^{3/2}}$$

   (A) $\frac{\sqrt{3}}{2}$ (B) $\frac{\sqrt{3}}{4} - \frac{1}{2}$ (C) $\frac{\sqrt{3}}{36} - \frac{1}{32}$ (D) $\frac{\sqrt{3}}{12}$ (E) $\frac{\sqrt{3}}{6} - \frac{1}{4}$ (F) $\frac{\sqrt{3}}{8} - \frac{1}{4}$

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6. Evaluate the following integral.

$$\int_3^\infty \frac{dx}{x^2 - 4x + 5}$$

   (A) $\frac{\sqrt{3}}{2} + \frac{1}{2}$ (B) $\frac{\sqrt{3}}{2} + \frac{1}{4}$ (C) $\frac{\sqrt{3}}{4} + \frac{1}{4}$ (D) $\frac{\pi}{2}$ (E) $\frac{\pi}{4}$ (F) diverges

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7. Calculate the volume of the solid obtained by rotating the region between the graphs of $y = \dfrac{1}{x^2 - 3x + 2}$ and $y = 0$ for $4 \le x \le 10$ around the y-axis.

(A) $2\pi \ln(8/25)$ (B) $2\pi \ln(8/9)$ (C) $4\pi \ln(8/5)$

(D) $4\pi \ln(8/3)$ (E) $2\pi \ln(16/5)$ (F) $2\pi \ln(16/3)$

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8. Find the area of the surface obtained by rotating the curve $y = \dfrac{x^3}{3}$, $0 \le x \le 1$ about the x-axis.

(A) $\dfrac{\pi(2\sqrt{2}-1)}{9}$ (B) $\dfrac{\pi(\sqrt{2}-2)}{3}$ (C) $\dfrac{\pi(2\sqrt{3}-1)}{9}$

(D) $\dfrac{\pi(\sqrt{3}-2)}{3}$ (E) $\dfrac{\pi(2\sqrt{2}-\sqrt{3})}{9}$ (F) $\dfrac{\pi(3\sqrt{2}-2\sqrt{3})}{3}$

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9. Evaluate the improper integral if possible:

$$\int_{2^{10}}^{\infty} \frac{1}{x^{1.1}}\, dx$$

(A) integral diverges (B) 5 (C) 50 (D) $\dfrac{1}{2}$ (E) $\dfrac{1}{5}$ (F) 25

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10. Find the equation for the line tangent to the curve defined by the parameterization $x = 1 + \dfrac{1}{t}$, $y = t^3 + 3$ (for $t > 0$) at the point $(x, y) = (2, 4)$.

(A) $y = -3x + 10$ (B) $y = -3x + 14$ (C) $y = 3x - 2$

(D) $y = 3x - 8$ (E) $y = -\dfrac{x}{3} + \dfrac{14}{3}$ (F) $y = -\dfrac{x}{3} + \dfrac{10}{3}$

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11. The curve given parametrically by

$$x = t^3 - t$$
$$y = \sqrt{3}(t^2 - 1)$$

passes through the origin for two different values of t, and hence contains a loop. What is the arc length of the loop?

(A) 4 (B) $4\sqrt{2}$ (C) $4\sqrt{3}$ (D) 3 (E) $3\sqrt{2}$ (F) $3\sqrt{3}$

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12. Find the area inside *one leaf* (i.e., one loop) of the graph of $r = 4\sin 4\theta$.

(A) $\frac{\pi}{8}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) π (E) 2π (F) 4π

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13. Find the limit.

$$\lim_{x\to 0} \frac{\cos x - 1}{e^x - x - 1}$$

(A) $1/e$ (B) π (C) -1 (D) 0 (E) 1 (F) no finite limit

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14. Find the limit of the sequence $\left\{\frac{1}{2}\ln(n^2+1) - \ln(2n+1)\right\}$.

(A) -2 (B) $-\ln 2$ (C) 0 (D) $\ln 2$ (E) 2 (F) sequence diverges

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15. Determine whether the series is convergent or divergent. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty} \frac{1+2^n}{6^n}$$

(A) $1/3$ (B) $1/2$ (C) $7/10$ (D) $3/2$ (E) $27/10$ (F) divergent

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16. How many of the following series converge?

$$\sum_{n=1}^{\infty} (\sqrt{2})^n, \qquad \sum_{n=1}^{\infty} \frac{1}{n}, \qquad \sum_{n=1}^{\infty} \left(\frac{e}{\pi}\right)^n, \qquad \sum_{n=1}^{\infty} \frac{2^n - 1}{3^n}, \qquad \sum_{n=1}^{\infty} \frac{n^2}{n^2+1}.$$

(A) none of these series converge (B) just one series converges

(C) two series converge (D) three series converge

(E) four series converge (F) all five series converge

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17. Which statement below is true about the following series?

- (I) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{1+n^2} = \frac{1}{2} - \frac{1}{5} + \frac{1}{10} - \dots$
- (II) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{1+n^2} = \frac{1}{2} - \frac{2}{5} + \frac{3}{10} - \dots$
- (III) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n \ln n}{1+n^2} = \frac{\ln 2}{5} - \frac{\ln 3}{10} + \frac{\ln 4}{17} - \dots$

(A) (I) diverges, (II) converges conditionally, (III) converges absolutely
(B) (I) diverges, (II) converges absolutely, (III) converges conditionally
(C) (I) and (III) converge conditionally, (II) converges absolutely
(D) (I) and (II) converge absolutely, (III) converges conditionally
(E) (I) and (III) converge absolutely, (II) converges conditionally
(F) (I) and (III) converge conditionally, (II) diverges

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18. Find the precise interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(3x-2)^n}{n3^n}.$$

(A) $(-1, 1]$
(B) $[-1, 1)$
(C) $\left(-\frac{1}{3}, \frac{5}{3}\right]$
(D) $\left[-\frac{1}{3}, \frac{5}{3}\right)$
(E) $\left[0, \frac{4}{3}\right]$
(F) $\left(0, \frac{4}{3}\right]$

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19. Which of the following is the beginning of the Maclaurin series for $\ln(1+x^3)$?

(A) $x^3 - \frac{x^6}{2} + \frac{x^9}{3} - \frac{x^{12}}{4} + \dots$
(B) $\frac{x^3}{3} - \frac{x^6}{6} + \frac{x^9}{9} - \frac{x^{12}}{12} + \dots$
(C) $x^3 - 2x^6 + 3x^9 - 4x^{12} + \dots$
(D) $1 + 2x^3 + 3x^6 + 4x^9 + \dots$
(E) $\frac{x^3}{3} + \frac{x^6}{6} + \frac{x^9}{9} + \frac{x^{12}}{12} + \dots$
(F) $x^3 + \frac{x^6}{2} + \frac{x^9}{3} + \frac{x^{12}}{4} + \dots$

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20. Let $F(x) = \int_0^x e^{-t^2} dt$. Which of the following is the beginning of the Maclaurin series for F?

(A) $x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$
(B) $x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{42} + \dots$
(C) $x - \frac{x^2}{2} + \frac{x^4}{6} - \frac{x^6}{24} + \dots$
(D) $1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \dots$
(E) $x - \frac{x^3}{3} + \frac{x^5}{15} - \frac{x^7}{105} + \dots$
(F) $x + \frac{x^3}{3} + \frac{x^5}{15} + \frac{x^7}{105} + \dots$