MATH 104 Fall 2013

COMMON FINAL EXAM

NAME: ______________________________

SECTION: ______________________________

My signature below certifies that I have complied with the University of Pennsylvania's *code of academic integrity* in completing this examination.

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Your signature

INSTRUCTIONS:

1. WRITE YOUR NAME at the top.
2. To obtain credit, you **MUST SHOW YOUR WORK**. You can receive partial credit based on your work, even if your final answer is wrong. Likewise, a right answer with poor or no work may not receive full credit.
3. There are to be NO calculators, cell phones (or any other kind of technology), or books, or notes during this exam. You are allowed **one double-sided handwritten 8.5 x 11in sheet of notes** during this exam.
4. You have 2 hours to complete the exam.
5. If you have a question, re-read carefully. These questions should not require clarification, and we will not give hints or explain a misunderstood concept. If you suspect an error in the question or a true ambiguity, please raise your hand and someone will help you directly.

PROBLEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	**TOTAL**
SCORE																

1. Compute the following improper integral: $\int_0^1 \frac{1}{\sqrt{1-x^2}} dx$.

(a) 0

(b) 1

(c) $\frac{\pi}{2}$

(d) π

(e) diverges

2. Set up, but do not evaluate, the integral for the area of the surface obtained by rotating the curve

$$y = \ln(x), \quad 1 \le x \le 4$$

about the x-axis.

(a) $\int_1^4 2\pi x\sqrt{1+x^{-2}}dx$

(b) $\int_1^4 2\pi \ln(x)\sqrt{1+x^{-2}}dx$

(c) $\int_1^4 2\pi \ln(x)\sqrt{1+\ln^2(x)}dx$

(d) $\int_1^4 2\pi x\sqrt{1+\ln^2(x)}dx$

(e) none of the above

3. Find the first few terms of the Maclaurin series for $f(x) = \int x \sin(-x) dx$

(a) $C + \frac{1}{3}x^3 + \frac{1}{3! \cdot 5}x^5 + \frac{1}{5! \cdot 7}x^7 + \ldots$

(b) $C - \frac{1}{4}x^2 + \frac{1}{4! \cdot 6}x^4 - \frac{1}{6! \cdot 8}x^6 + \ldots$

(c) $C - \frac{1}{3}x^3 + \frac{1}{3! \cdot 5}x^5 - \frac{1}{5! \cdot 7}x^7 + \ldots$

(d) $C + x^3 - \frac{1}{3! \cdot 5}x^3 - \frac{1}{5!}x^5 + \ldots$

(e) $C + \frac{1}{4}x^2 + \frac{1}{4! \cdot 6}x^4 - \frac{1}{6! \cdot 8}x^6 + \ldots$

(f) $C + \frac{1}{3}x^3 - \frac{1}{3!}x^5 + \frac{1}{5!}x^7 + \ldots$

4. Find the volume of the solid obtained by rotating the region bounded by the curves

$$y = x \cdot \cos x \text{ and the } x\text{-axis for } 0 \leq x \leq \frac{\pi}{2}$$

about the y-axis.

(a) 4π

(b) $\frac{\pi^3}{2} - 4\pi$

(c) 1

(d) $2\pi^2$

(e) $\frac{\pi^2}{4} - 6\pi$

(f) $4\pi^3$

5. Evaluate $\int_{1/2}^{1} \frac{y+4}{y^2+y} dy$.

(a) $\ln\left(\frac{27}{4}\right)$

(b) $\ln\left(\frac{8}{3}\right)$

(c) $\frac{1}{2} \ln\left(\frac{8}{3}\right)$

(d) $\frac{1}{2} + 3(\ln 3 - \ln 2)$

(e) $\frac{9}{2} \ln 3 - 5 \ln 2$

6. The probability density function $f(x)$ is equal to ke^{-3x} for $x \geq 0$ and 0 for $x < 0$. Determine the value of k and the mean μ.

   (a) $k = 3, \mu = \frac{1}{3}$

   (b) $k = 3, \mu = -\frac{1}{3}$

   (c) $k = 1, \mu = \frac{1}{9}$

   (d) $k = 1, \mu = -\frac{1}{9}$

7. Solve $y' = \dfrac{\cos x}{y^2}$ with initial value $y\left(\dfrac{\pi}{2}\right) = 1$.

(a) $y = \sqrt[3]{3\sin x - 2}$

(b) $y = \sqrt[3]{3\sin x + 1}$

(c) $y = \sqrt[3]{\sin x}$

(d) $y = \sqrt[3]{3\sin x}$

8. Suppose that $\sum_{n=0}^{\infty} a_n$ converges to 2. Then $\sum_{n=0}^{\infty} e^{a_n}$

(a) converges to 1

(b) converges to 2

(c) converges to 2^e

(d) converges to e^2

(e) diverges

9. Which of these quantities is closest to $\sin(1)$?

(a) $\frac{4}{5}$

(b) $\frac{5}{6}$

(c) $\frac{101}{120}$

(d) $\frac{51}{60}$

(e) $\frac{13}{15}$

(f) 1

10. Determine the interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{n!}{4^n (2n)!}(x-4)^n$$

(a) $[3, 5)$

(b) $[3, 5]$

(c) $(3, 5]$

(d) $(3, 5)$

(e) $(-\infty, \infty)$

(f) $\{4\}$

11. Find the Taylor polynomial $P_4(x)$ of order 4 for $e^{\sin x}$ at $x = 0$ and evalaute it at $x = 1$. Then $P_4(1) =?$

(a) $\frac{1}{8}$

(b) 0

(c) 9

(d) $\frac{1}{3}$

(e) 4

(f) $\frac{19}{8}$

12. A thin plate of constant density $\delta = 2$ is bounded between the graphs $y = e^{3x}$ and the lines $x = 0$, $x = 1$ and $y = 0$. Find the moment about the y-axis of the plate.

(a) $\frac{1}{4}(3 + 2e^3)$

(b) $\frac{1}{9}(2 + e^3)$

(c) $\frac{2}{9}(1 + 2e^3)$

(d) $\frac{3}{7}(1 + e^3)$

(e) $\frac{1}{3}(2 + 2e^3)$

13. Which statement is true for the following series

I. $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n^3 \cdot \sqrt{n+1}}$ II. $\displaystyle\sum_{n=1}^{\infty} \frac{7^n}{2^n - 3n + 2}$

III. $\displaystyle\sum_{n=2}^{\infty} \frac{\cos(n\pi)}{\ln n}$ IV. $\displaystyle\sum_{n=2}^{\infty} \frac{n^2 \cdot \sin(n)}{2^n}$

(a) All four series diverge

(b) All converge

(c) II and III converge

(d) I and II converge

(e) I and IV converge

(f) I and III converge

(g) II and IV converge

(h) III and IV converge

(i) I, II and III converge

(j) I, III and IV converge

(k) II, III and IV converge

(l) I, II and IV converge

14. Which equation best models the following statement? You may assume that P stands for population and t for time.

> "The percentage growth rate of a population remains constant at around 3% per year."

(a) $P(t) = P(0) + 0.03tP(0)$

(b) $P(t) = (1.03)^t$

(c) $P(t) = (1.03)^t P(0)$

(d) $P'(t) = 0.03$

(e) $P'(t) = 0.03P(t)$

(f) $P'(t) = 3\%$

15. The arc length of the portion of the curve $y = \dfrac{e^x + e^{-x}}{2}$ from the point where $x = 0$ to the point where $x = \ln(4)$ is

(a) $\dfrac{11}{4}$

(b) $\dfrac{17}{4}$

(c) $\dfrac{11}{8}$

(d) $\dfrac{15}{8}$

(e) $\dfrac{17}{8}$

(f) infinite