MATH 104 Spring 2013: Calculus

FINAL EXAM

NAME: ____

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My signature below certifies that I have complied with the University of Pennsylvania's *code of academic integrity* in completing this examination.

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Your signature

INSTRUCTIONS:

1. WRITE YOUR NAME at the top.
2. TRANSFER YOUR ANSWER CHOICES (letter only) into the second row of the table at the bottom of this page.
3. To obtain credit, you **MUST SHOW YOUR WORK**. You will receive partial credit based on your work, even if your final answer is wrong. Likewise, a right answer with poor or no work will not receive full credit.
4. There are to be NO calculators, cell phones (or any other kind of technology), or books, or notes during this exam. You are allowed **one double-sided handwritten 8.5 x 11in sheet of notes** during this exam.
5. You have 2 hours to complete the exam. Do the exam quickly, but read carefully! You gain nothing by doing a different problem than what is asked.
6. If you have a question, re-read carefully. These questions should not require clarification, and we will not give hints or explain a misunderstood concept. If you suspect an error in the question or a true ambiguity, please raise your hand and someone will help you directly.
7. Best wishes.

SCORE:

PROBLEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	**TOTAL**
ANSWERS																

Do not write below this line.

SCORE																

PROBLEM 1:

Find the volume of the solid of revolution obtained by revolving the region bounded by graphs of functions $f(x) = x^3$, $x = 0$, $y = 8$ around y-axis.

(a) $\frac{96\pi}{5}$ (b) $\frac{94\pi}{5}$ (c) $\frac{92\pi}{9}$ (d) $\frac{98\pi}{5}$ (e) $\frac{89\pi}{7}$ (f) $\frac{16}{5}$

PROBLEM 2: Which of the following integrals can be used to compute the volume of the solid of revolution obtained by revolving the region bounded above by $y = 2 - \frac{x^4}{2}$ and below by $y = -6$ around the line $x = 5$.

Justify your answer with a picture.

(a) $\int_{-1}^{1} 2\pi \left(2 - \frac{x^4}{2}\right)(5 - x)\, dx$ (b) $\int_{-2}^{2} 2\pi \left(2 - \frac{x^4}{2}\right)(5 - x)\, dx$

(c) $\int_{-2}^{2} 2\pi \left(4 - \frac{x^4}{2}\right)(7 - x)\, dx$ (d) $\int_{-2}^{2} 2\pi \left(2 - \frac{x^4}{2}\right) x\, dx$

(e) $\int_{-2}^{2} 2\pi \left(8 - \frac{x^4}{2}\right)(5 - x)\, dx$ (f) $\int_{-2}^{3} 2\pi \left(4 - \frac{x^5}{2}\right)(5 - x)\, dx$

PROBLEM 3: Find the arc length of the function $f(x)$ whose derivative is

$$f'(x) = \sqrt{x^2(\ln x)^2 - 1},$$

between $x = 1$ and $x = e$.

(a) $\frac{e^3+1}{4}$ (b) $\frac{e^2+1}{5}$ (c) $\frac{e^2+1}{4}$ (d) $\frac{e^2+3}{4}$ (e) $\frac{2e^2+1}{4}$ (f) $\frac{2e^3+1}{5}$

PROBLEM 4: Evaluate the following integral

$$\int_0^2 \sqrt{4 - x^2}\, dx.$$

(a) π (b) 2π (c) $2\pi + 1$ (d) 3π (e) $3\pi + 3$ (f) 1

PROBLEM 5: Compute the following integral

$$\int \frac{5+x}{x^2+x-6}\,dx.$$

(a) $\frac{3}{5}\ln|x-2| - \frac{1}{5}\ln|x+3| + C$ (b) $\frac{7}{5}\ln|x-3| - \frac{2}{5}\ln|x+3| + C$

(c) $\frac{3}{5}\ln|x-2| - \frac{2}{5}\ln|x+3| + C$ (d) $\frac{7}{5}\ln|x-2| - \frac{1}{5}\ln|x+3| + C$

(e) $\frac{7}{5}\ln|x-2| - \frac{2}{5}\ln|x+3| + C$ (f) $\frac{3}{5}\ln|x-3| - \frac{2}{5}\ln|x+3| + C$

PROBLEM 6: Evaluate the following improper integral or show that it does not converge:

$$\int_0^1 x^{-\frac{2}{5}}\,dx = \int_0^1 \sqrt[5]{\frac{1}{x^2}}\,dx.$$

(a) 2 (b) $\frac{10}{7}$ (c) 4 (d) $\frac{5}{3}$ (e) $\frac{11}{3}$ (f) The integral diverges.

PROBLEM 7: What is the centroid of the region bounded by the curves $y = x^2$ and $y = 8 - x^2$?

Hint: draw a picture of this region as your first step.

(a) $(-2, 3)$ (b) $(2, 5)$ (c) $(-1, 4)$ (d) $(0, 4)$ (e) $(0, 3)$ (f) $(1, 4)$

PROBLEM 8: Find the constant C so that the function

$$f(x) = \begin{cases} C\sqrt{x-1}, & 1 \le x \le 2; \\ 0, & \text{otherwise.} \end{cases}$$

is a probability density function, and then compute its mean m.

(a) $C = 4,\ m = \frac{9}{5}$ (b) $C = \frac{3}{2},\ m = \frac{3}{5}$ (c) $C = 2,\ m = 1$

(d) $C = 3,\ m = \frac{8}{5}$ (e) $C = \frac{3}{2},\ m = \frac{7}{5}$ (f) $C = \frac{3}{2},\ m = \frac{8}{5}$

PROBLEM 9: Solve the initial value problem:

$$\begin{aligned} \frac{dy}{dx} &= \frac{y^2}{x^2+1} \\ y(0) &= 1. \end{aligned}$$

(a) $y = \dfrac{-1}{\arctan x - 1}$ (b) $y = \dfrac{1}{\arctan x + 1}$ (c) $y = \dfrac{2}{\arctan x - 1}$

(d) $y = \dfrac{x}{\arctan x - 1}$ (e) $y = \dfrac{2x}{\arctan x + 1}$ (f) $y = \dfrac{1}{\arctan x}$

PROBLEM 10: Find the general solution of

$$xy' = y + \frac{x^2}{x+1}.$$

(a)	$x \ln\|x+1\| + x + C$	(b)	$x \ln\|x+1\| + Cx^2$	(c)	$x \ln\|x+1\| + Cx$
(d)	$x^2 \ln\|x+1\| + Cx$	(e)	$x \ln\|x+1\| + x^2 + C$	(f)	$Cx \ln\|x+1\| + x$

PROBLEM 11: Find the limit as n goes to infinity $\lim_{n\to\infty} a_n$ of the sequence $\{a_n\}_{n\geq 0}$

$$a_n = \cos\left(\frac{\pi}{2} + \frac{1}{n^2+1}\right).$$

(a) 0 (b) 1 (c) -1 (d) $\frac{1}{2}$ (e) $\frac{\sqrt{2}}{2}$ (f) 2

PROBLEM 12: Determine whether the following series converge or diverge

$$\text{(i)} \quad \sum_{n=1}^{\infty} (\ln(2n) - \ln n) \quad \text{(ii)} \quad \sum_{n=1}^{\infty} \frac{1}{n^{\pi}} \quad \text{(iii)} \quad \sum_{n=1}^{\infty} \frac{(\cos(n))^2}{\sqrt{n^5}}$$

You must explain your reasoning for each series, even if you can deduce the answer by process of elimination.

(a)	All series converge	(b)	(i) and (iii) diverge; (ii) converges
(c)	(i) and (ii) diverge; (iii) converges	(d)	All series diverge
(e)	(ii) and (iii) converge; (i) diverges	(f)	(ii) converges; (i) and (iii) diverge

PROBLEM 13: Find the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{n}{5^n}(x+3)^n.$$

(a) $(-2, 8)$ (b) $(-8, 2)$ (c) $(-8, 2]$ (d) $[-2, 8]$ (e) $(-\infty, \infty)$ (f) $[\frac{14}{5}, \frac{16}{5})$

PROBLEM 14: Compute the following limit:

$$\lim_{n\to\infty}\sum_{k=0}^{n}\frac{1}{k!}=\lim_{n\to\infty}\left(1+1+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\ldots+\frac{1}{n!}\right).$$

(a) π (b) 1 (c) e (d) $\frac{1}{2}$ (e) $e+\pi$ (f) 2

PROBLEM 15: For what values of x can $\sin x$ be approximated by $x - \frac{x^3}{3!}$ with an error strictly less than $\frac{1}{10}$?

(a) $-\sqrt[7]{12} < x < \sqrt[7]{12}$ (b) $-1 < x < 1$ (c) $-\sqrt[5]{10} < x < \sqrt[5]{10}$

(d) $-2 < x < 2$ (e) $-\sqrt[5]{12} < x < \sqrt[5]{12}$ (f) $-\sqrt[5]{13} < x < \sqrt[5]{13}$

EXTRA SPACE