Math 104 Final Exam — Spring 2010

1. Compute the area of the region in the first quadrant bounded by the curves $x = \sqrt{2-y}$, $y = x^2$, and the y-axis.

(a) 1/6 (b) 5/6 (c) 7/6 (d) 4/3 (e) 3/4 (f) 21/12

2. Find the volume of the solid obtained by rotating the region bounded by the curves $y = 1/x$ and the x-axis between $x = 1$ and $x = 2$ about the x-axis.

(a) π (b) $\pi/2$ (c) $\pi/3$ (d) $\pi/4$ (e) $2\pi/3$ (f) $3\pi/4$

3. Find the volume of the solid obtained by rotating the region bounded by the curves $y = x$ and $y = x^2$ about the y-axis.

(a) $\pi/2$ (b) $\pi/3$ (c) $\pi/4$ (d) $\pi/5$ (e) $\pi/6$ (f) $\pi/7$

4. Evaluate $\displaystyle\int_1^{e^3} \frac{\sqrt{\ln x}}{x}\,dx$.

(a) $2\sqrt{3}$ (b) $e^2 - e^{-2}$ (c) $\frac{e^2}{3}$ (d) $e^2 - 1$ (e) $\sqrt{\ln e^4} - 1$ (f) 3.46

5. Evaluate $\displaystyle\int_0^{\pi/2} x^2 \sin x\,dx$.

(a) $\pi/4$ (b) -1 (c) $\pi - 2$ (d) $\ln(\pi/2)$ (e) $\pi - e$ (f) 1.14

6. Evaluate $\displaystyle\int_0^{1/2} \cos^3(\pi x)\,dx$.

(a) $\sqrt{2} + \pi$ (b) $\pi/2$ (c) $\frac{\pi}{4} - e + \frac{1}{2}$ (d) $\sqrt{\pi}$ (e) $\ln 2$ (f) $\frac{2}{3\pi}$

7. Evaluate $\displaystyle\int_0^1 \frac{4x}{(x+1)(x^2+1)}\,dx$.

(a) $\pi/4 - 1/2$ (b) $\pi/2 - \ln 2$ (c) 32/51 (d) $\pi/8$ (e) $e/9$ (f) 0.28

8. Evaluate the improper integral $\displaystyle\int_2^{\infty} \frac{1}{(x-1)^3}\,dx$.

(a) $\frac{1}{2}$ (b) $\frac{\pi}{2} - e$ (c) $\frac{\pi}{4}$ (d) $\sqrt{\pi}$ (e) $\ln 2 + \frac{1}{3}$ (f) divergent

9. Find the arclength of the part of the curve $y = \frac{2}{3}(x-4)^{3/2}$ between the points $(7, 2\sqrt{3})$ and $(12, \frac{32}{3}\sqrt{2})$.

(a) 19 (b) 65/2 (c) 55/2 (d) 55 (e) 38/3 (f) 19/2

10. An artist is designing a wine glass in a flower shape, which can be generated by rotating the region bounded by $y = \sqrt{x}$ and $x = y$, between $x = 0$ and $x = 1$, about x-axis. What is the surface area (which contains both the inside and the outside surfaces) of such a glass?

(a) $\left(\frac{8\sqrt{2}-4}{3} + \sqrt{2}\right)\pi$ (b) $\left(\frac{8\sqrt{2}-4}{3} + \sqrt{5}\right)\pi$ (c) $\left(\frac{8\sqrt{2}-4}{3} + 1\right)\pi$

(d) $\left(\frac{5\sqrt{5}-1}{6} + \sqrt{2}\right)\pi$ (e) $\left(\frac{5\sqrt{5}-1}{6} + \sqrt{5}\right)\pi$ (f) $\left(\frac{5\sqrt{5}-1}{6} + 1\right)\pi$

11. What is the x-coordinate of the centroid of the region bounded by the graph of $y = \sqrt[3]{x}$, the line $x = 8$, and the x-axis?

(a) 8 (b) $\dfrac{16}{7}$ (c) $\dfrac{32}{7}$ (d) $\dfrac{2}{3}$ (e) 4 (f) $\dfrac{224}{9}$

12. Consider the initial value problem $\frac{dy}{dt} - y = 1$, with $y(0) = 3$. Find $y(1)$.

(a) e (b) $e - 1$ (c) $3e$ (d) $3e - 1$ (e) $4e$ (f) $4e - 1$

13. The size $P(t)$ of a certain population at time t satisfies the differential equation $\dfrac{dP}{dt} = \dfrac{P}{2}\left(1 - \dfrac{P}{2000}\right)$. At t=0 the population is 400. What is the behavior of $P(t)$ as $t \to \infty$?

(a) $P(t) \to 0$ (b) $P(t) \to \infty$ (c) $P(t) \to 4000$ (d) $P(t) \to 2000$
(e) $P(t) \to 800$ (f) $P(t)$ oscillates between 200 and 4000.

14. Determine if the sequence $a_n = \dfrac{\sqrt{n+1} - \sqrt{n}}{\sqrt{n+1} + \sqrt{n}}$ converges or diverges. If it converges, find the limit.

(a) 2 (b) $\dfrac{1}{2}$ (c) $\sqrt{2}$ (d) $\dfrac{1}{\sqrt{2}}$ (e) 0 (f) The sequence is divergent.

15. Determine if the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n-1)}$ converges or diverges. If it converges, find the sum.

(a) $\dfrac{1}{6}$ (b) $\dfrac{1}{2}$ (c) 1 (d) $\dfrac{3}{4}$ (e) $\dfrac{3}{2}$ (f) The series is divergent.

16. The series $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{n!}$

(a) converges because the terms approach 0.
(b) diverges because the terms do not approach 0.
(c) converges by the alternating series test.
(d) diverges by the alternating series test.
(e) converges by the comparison test.
(f) diverges by the geometric series test.

17. The series $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{(n+3)2^{2n}}{3^{n+100}}$

(a) converges absolutely by the ratio test.
(b) converges conditionally (but not absolutely) by the ratio test.
(c) diverges by the ratio test.
(d) converges absolutely by comparison with $\sum_{n=1}^{\infty} \frac{1}{3^n}$.
(e) converges conditionally (but not absolutely) by comparison with $\sum_{n=1}^{\infty} (-1)^n \frac{n+3}{3^n}$.
(f) diverges by comparison with $\sum_{n=1}^{\infty} (-1)^n 4^n$.

18. The power series $\sum_{n=0}^{\infty} \frac{(x-3)^n}{2^n}$ converges

(a) to the function $2/(5-x)$ precisely on the interval $1 < x < 5$.
(b) to the function $2/(5-x)$ precisely on the interval $-3 < x < 3$.
(c) to the function $5/(2-x)$ precisely on the interval $0 \le x \le 2$.
(d) to the function $5/(2-x)$ precisely on the interval $0 \le x \le 6$.
(e) only at $x = 0$.
(f) only at $x = 3$.

19. The Maclaurin series for the function $x\cos(2x)$ is

(a) $\frac{2x^2}{2!} - \frac{2x^4}{4!} + \frac{2x^6}{6!} - \frac{2x^8}{8!} + \cdots$.

(b) $x - \frac{2x^2}{2!} + \frac{2^2x^3}{3!} - \frac{2^3x^4}{4!} + \cdots$.

(c) $1 + 2x + \frac{2^2x^2}{2!} + \frac{2^3x^3}{3!} + \cdots$.

(d) $1 - \frac{x^2}{2\cdot 2!} + \frac{x^4}{2\cdot 4!} - \frac{x^6}{2\cdot 6!} + \cdots$.

(e) $x - \frac{2^2x^3}{2!} + \frac{2^4x^5}{4!} - \frac{2^6x^7}{6!} + \cdots$.

(f) $x - 2x^2 + 2^2x^3 - 2^3x^4 + \cdots$.

20. Consider the polynomial $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$ as an approximation to e^x on the interval $-2 \le x \le 2$. What is the best bound on the error for this estimate that is given by Taylor's inequality?

(a) $1/24$ (b) $e/12$ (c) $2e^2/3$ (d) $e^3/4$ (e) $3e^4/2$ (f) e^5