1. Solve the initial-value problem. $\frac{dx}{dt} + 2tx = x,\ x(0) = 5$. Use your solution to compute $x(3)$.

a) $5e^{-6}$ b) $5e^{6}$ c) $6e^{5}$ d) 3 e) –10

Ans: a

2. The volume of the solid generated by revolving the region bounded by the curves $x = y^2$ and $y = x - 2$ about the y-axis

a) $\frac{20\pi}{3}$ b) $\frac{72\pi}{5}$ c) $\frac{42\pi}{5}$ d) $\frac{13\pi}{2}$ e) $\frac{32\pi}{5}$ f) $\frac{212\pi}{15}$

Ans b

3. Find the volume of the solid generated by rotating about the y-axis the region enclosed by $y = \sin x$ and the x-axis from $x = 0$ to $x = \pi$.

(A) $\frac{\pi^2}{2}$ (B) $\frac{\pi}{2}$ (C) 4

(D) 2 (E) $4\pi^2$ (F) $2\pi^2$

4, Which of the following statements is true about the series $\sum_{n=0}^{\infty} (-1)^n \cos\left(\frac{1}{n}\right)$? Be sure the work you show justifies your choice.

a) the series is absolutely convergent
b) the series is conditionally convergent
c) the series is divergent

Ans: c

5. What is the interval of convergence of the series $\sum_{n=0}^{\infty} \frac{n^3 x^{3n}}{n^4 + 1}$

a) the series converges only at $x = 0$
b) the series converges for all x
c) the series diverges for $x \neq 0$
d) the series converges on (-1, 1]
e) the series converges on [-1, 1)
f) the series converges on [-1, 1]

6. Find the Taylor series about $a = 0$ for $\frac{1}{1+2x^2}$.

(A) $\sum_{n=0}^{\infty}(-1)^n \cdot 2^{2n} \cdot x^{2n} = 1 - 4 \cdot x^2 + 16 \cdot x^4 - 64 \cdot x^6 + \ldots$

(B) $\sum_{n=0}^{\infty}(-1)^n \cdot 2^{n} \cdot x^{2n} = 1 - 2 \cdot x^2 + 4 \cdot x^4 - 8 \cdot x^6 + \ldots$

(C) $\sum_{n=0}^{\infty} 2^{2n} \cdot x^{2n} = 1 + 4 \cdot x^2 + 16 \cdot x^4 + 64 \cdot x^6 + \ldots$

(D) $\sum_{n=0}^{\infty} 2^{n} \cdot x^{2n} = 1 + 2 \cdot x^2 + 4 \cdot x^4 + 8 \cdot x^6 + \ldots$

(E) $\sum_{n=0}^{\infty} x^{2n} = 1 + x^2 + x^4 + x^6 + \ldots$

(F) $\sum_{n=0}^{\infty}(-1)^n x^{2n} = 1 - x^2 + x^4 - x^6 + \ldots$

Ans: b

7. What is the length of the part of the curve $y = x^2 - \frac{\ln x}{8}$ between the points (1,1) and $(e^2, e^2 - \frac{1}{8})$?

(a) $y = e^3 - \frac{1}{2}$ (b) $y = \frac{1}{2}e^3 - \frac{3}{8}$ (c) $y = \frac{1}{3}e^2 + \frac{1}{8}$

(d) $y = e^2 - \frac{7}{8}$ (e) $y = \frac{1}{2}e^2 - \frac{3}{8}$ (f) $y = \frac{1}{3}e^3 - \frac{5}{8}$

Ans d

8. Integrate: $\int_0^1 \frac{3x+2}{x^2-4}\,dx$

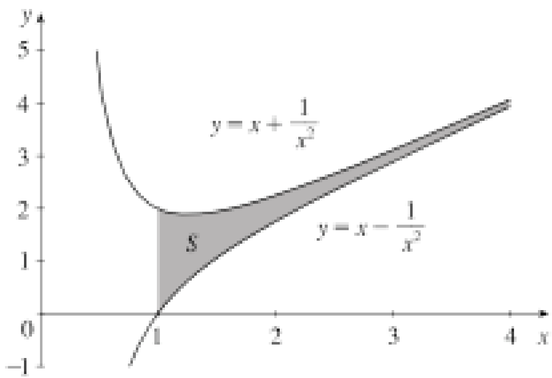
(A) -2 (B) $-\ln 3$ (C) $-\ln 2$ (D) 0 (E) $\ln 2$ (F) 2

a) -2 b) -3ln2 + ln3 c) ln2 d) $\pi/4$ e) 0 f) ln(3)

Ans: b

9

Consider the region S bounded by the curves $y = x + \frac{1}{x^2}$ and $y = x - \frac{1}{x^2}$ for $x \geq 1$.



Is the area S finite or infinite? If finite, what is the area?

a) the integral diverges, so the area is not finite
b) area = 0
c) area = 1
d) area= 2
e) area = π
f) area = 4

Ans. d

10. Evaluate the following integral: $\int_0^1 \frac{3\ln 4x}{\sqrt{x}}\,dx$

a) 12ln2 – 12 b) 0 c) 12(1 – ln2) d) 0 e) 12 f) divergent

Ans a

11. Evaluate the integral or show it is divergent:

$$\int_1^\infty \frac{dx}{x\ln x}$$

a) 0 b) 1 c) e d) e^e e) ln(4) f) divergent

Ans: f

12. The base of a solid is the region enclosed by the ellipse $4x^2 + y^2 = 1$. If all the plane crossections perpendicular to the x axis are semicircles, compute the volume of the solid.

a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{2}$ e) $\frac{2\pi}{3}$ f) $\frac{3\pi}{4}$

Ans c

13. Which of the following statements about the alternating series $\sum_{n=1}^{\infty}(-1)^n a_n$

where $a_n = \frac{n}{1+n^2}$ is true?

a) the series is absolutely convergent
b) the series is conditionally convergent
c) the series is divergent

Ans: b

14. Evaluate the integral $\int_0^1 \frac{x^3}{\sqrt{1-x^2}}dx$

a) π/4 b) π/2 c) π d) 2/3 e) ¾ f) 1

Ans: d

15. Solve the differential equation. $7yy' = 5x$

a. $7x^2 - 5y^2 = C$ b. $5x^2 + 7y^2 = C$ c. $5x^2 - 7y^2 = C$ d. $7x^2 + 5y^2 = C$ e. $5x^2 + 7y^2 = 12$

Ans: c

16. Find the average value of $f(x) = \sin^2 x \cos^3 x$ over the interval [–π, π]

a) π b) 0 c) $\frac{\pi}{5}$ d) $\frac{\pi}{6}$ e) $\frac{\pi}{12}$ f) $\frac{1}{2\pi}$

Ans: b

17. Consider the sequence defined by $a_n = \frac{(-1)^n + n}{(-1)^n - n}$. Does this sequence converge and, if it does, to what limit?

a) yes, to –1 b) yes, to 0 c) yes, to 1 d) yes, to 2 e) yes to π f) diverges

Ans a

18. Find the area of the surface obtained by rotating the curve $y = \frac{1}{4}x^2 - \frac{1}{2}\ln x, 1 \le x \le 2$ about the y-axis.

a. $\frac{101\pi}{2}$ b. $\frac{99\pi}{2}$ c. 48π d. 24π e. 12π f) none of these

Ans: f

19. Use the Alternating Series Estimation Theorem or Taylor's Inequality to estimate the largest range of values of x for which the given approximation is accurate to within the stated error.

$\cos x \approx 1 - \frac{x^2}{2} + \frac{x^4}{24}$, $|error| < 0.08$

a) only for $x = 0$ b) $-1 \le x \le 1$ c) $-\sqrt[6]{7.2} \le x \le \sqrt[6]{7.2}$ d) $-2 \le x \le 2$
e) $-\sqrt[6]{57.6} \le x \le \sqrt[6]{57.6}$ f) $-\pi \le x \le \pi$

Ans. e

20. Find a series representation for $\int \frac{e^x}{x} dx$.

a) $\sum_{n=0}^{\infty} \frac{x^n}{(n+1)!} + C$
b) + C
c) $\ln|x| + \sum_{n=1}^{\infty} \frac{x^n}{n \cdot n!} + C$
d) $\ln|x| + \sum_{n=1}^{\infty} \frac{n+1}{n!} x^n + C$
e) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n} x^n + C$
f) $\ln|x| + \sum_{n=1}^{\infty} \frac{1}{(n+1)!} x^{n+1} + C$

Ans: c