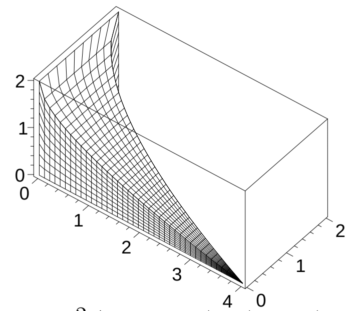
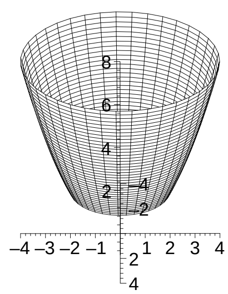
Math 104 Final Exam Fall 2002

1. A scientist collects data that relate two variables, x and y. Instead of plotting y as a function of x, she plots $\log_2 y$ as a function of $\log_2 x$, and gets a line whose slope is 3 and whose intercept on the vertical axis is 5. What equation describes y as a function of x?

2. Find the volume of the solid that lies between the planes perpendicular to the x-axis at $x = 0$ and $x = 4$, and whose cross-sections perpendicular to the x-axis are squares one side of which runs from the x-axis to the curve $y = 2 - \sqrt{x}$.



3. The arc of the parabola $y = x^2/2$ from (2,2) to (4,8) is rotated around the y-axis. Find the surface area of the resulting surface.



4. Solve the initial-value problem:

$$(x+1)\frac{dy}{dx} + 2y = x, \qquad y(0) = 1$$

5 (a). A super-fast-growing bacteria reproduces so quickly that the rate of production of new bacteria is proportional to the *square* of the number already present. Write the differential equation that describes the growth of the number of bacteria, and then give the general solution of this differential equation.

5 (b). Suppose a sample of the bacteria from problem 5 (a) starts with 10 bacteria, and after 2 hours there are 20 bacteria. Write the formula for the number of bacteria after t hours. How long will it take until there are (theoretically) an *infinite* number of bacteria?

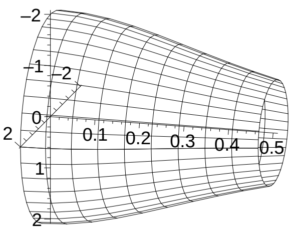
6. Calculate $\int_0^1 x^3 \ln x \, dx$, if it converges.

7. Calculate $\int \frac{x}{x^2 - 3x + 2} \, dx$.

8. Evaluate $\displaystyle\int_0^{\pi/2} \sin^3\theta\cos^2\theta\,d\theta$.

9. Sketch the graph of $y = xe^{-x}$ on the interval $[0,\infty)$. Be sure to indicate "interesting" points on the graph as well as any asymptotes it has.

10. Calculate the volume of the solid obtained by rotating the part of the graph of $y = \dfrac{2}{4x^2+1}$ for $x \in [0, \frac{1}{2}]$ around the x-axis.



11. Investigate the convergence/divergence of the improper integral $\displaystyle\int_0^{\infty} \frac{dx}{x^2(1+e^x)}$. (Do not attempt to evaluate the integral.)

12. Find the limit of the sequence $\{\sqrt[n]{3^n + 7^n}\}$ or else explain why it does not converge.

13. Investigate the convergence (absolute/conditional/divergence) of the series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \ln n}{1+n}$.

14 (a). Use the first three non-zero terms of an appropriate series to give an approximation of

$$\int_0^{1/2} e^{-x^2}\,dx.$$

14 (b). Give (with explanation) an estimate of the error (difference between your approximation and the actual value of the integral) in problem 14 (a).

15 (a). Find the center and radius of convergence of the power series

$$\sum_{n=2}^{\infty} \frac{(-1)^n(1+n)}{2n}(x-5)^n.$$

(In other words, find the largest *open* interval on which the series converges).

15 (b). Investigate the convergence (absolute/conditional/divergence) of the series in 15 (a) at the endpoints of its interval of convergence.

16 (a). Write the second-degree Taylor polynomial for $f(x) = \sqrt{x}$ centered at $a = 100$.

16 (b). Use the polynomial from 16 (a) to estimate $\sqrt{101}$. Also, give an estimate of the error.

17. Calculate $\displaystyle\lim_{x\to\infty} \frac{1}{x}\int_0^x \arctan t\,dt$.

18. Does the series $\displaystyle\sum_{n=2}^{\infty} \frac{\log_n(n!)}{n^3}$ converge or diverge? Explain your answer.