

Final exam

Math 103
5/11/2015

Name: _____

ID: _____

“My signature below certifies that I have complied with the University of Pennsylvania’s Code of Academic Integrity in completing this exam:”

Signature: _____

Read all of the following information before starting the exam:

- Check your exam to make sure all 12 pages are present.
- The exam questions are not in a particular order. If you get stuck, move on to the next problem.
- One 8.5×11 sheet of handwritten notes (front and back) is allowed.
- DNE means “does not exist”.
- You will be provided with scratch paper; you must turn it in with your exam.
- You **MUST** show work to receive credit. Even if you choose the correct answer for a multiple choice problem, you will not receive credit if I don’t see how you arrived at your answer.
- Good luck!

1	15		7	15	
2	15		8	15	
3	15		9	10	
4	10		10	20	
5	20		11	10	
6	15				
Total	160				

1. Compute each limit or explain why it does not exist:

1. $\lim_{x \rightarrow \infty} \sqrt{x-1} - \sqrt{x}$

2. $\lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{x}$

3. $\lim_{x \rightarrow 2} \frac{x^2 - 4}{(x-2)^2}$

2. The curves $y = x^2$, $x + y = 2$ and $y = 0$ bound a finite region in the plane having points below the graphs of $y = x^2$, $x + y = 2$ and above the x -axis.

1. Carefully sketch the region.
2. Represent the area of the region as an integral with respect to y .
3. Compute the integral to find the area.

3. Water is flowing into an inverted conical tank (i.e., the tank has the shape of an upside-down cone). The tank is 10 feet deep and has a radius of 5 feet at the top. The water is flowing at a rate of $2\frac{\text{ft}^3}{\text{sec}}$. (*Hint:* The volume of a cone of radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$.)

1. At what rate is the water level rising in the tank?
2. If the tank starts out empty, how long will it take to fill?

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- _____ 4. What is the value of $\lim_{x \rightarrow 0^+} x \ln x$? Show your work.
- | | |
|--------------|-------------|
| a. -1 | d. 0 |
| b. $-\infty$ | e. DNE |
| c. $-e$ | f. ∞ |

5.

1. Find the derivative of each function below:

(a) 9^{2t}

(b) $\ln(\arccos x)$

(c) $\left(\frac{\sqrt{x}}{1+x}\right)^2$

2. Find the following antiderivatives:

(a) $\int \frac{(\arctan x)^2}{1+x^2} dx$

(b) $\int \frac{(t+1)^2-1}{t^4} dt$

6. You are 2 miles offshore in a boat and wish to reach a coastal village 6 miles down a straight shoreline from the point nearest the boat. You can row 2 mi/h and walk 5 mi/h. Where should you land your boat to reach the village in the least amount of time? (Don't worry about checking the endpoints; in this problem, the absolute minimum occurs at a critical point.)

7. Let f be an arbitrary continuous function defined everywhere. For each of the following statements, say whether it is necessarily true or could be false. If it could be false, explain why (for instance, by drawing a picture or giving a counterexample).

a. If $b \leq c$, then $\int_a^b f(x) dx \leq \int_a^c f(x) dx$.

b. $F(t) := \int_0^t |f(x)| dx$ is an increasing function.

c. $\int_0^1 f(x) \cdot g(x) dx = \left(\int_0^1 f(x) dx \right) \cdot \left(\int_0^1 g(x) dx \right)$.

d. If $\int_a^b f(x) dx$ exists, then f is differentiable on (a, b) .

8.

1. Represent the following limit as a definite integral:

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(2 \left(1 + \left(\frac{4k}{n} \right) \right) - 1 \right)^{-\frac{1}{2}} \left(\frac{4}{n} \right).$$

(Hint: It is a limit of right-endpoint Riemann sums.)

2. Compute the value of the integral that you found.

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9. The point $(1, 1)$ is on the graph of $xy^2 + yx^2 = 2$. What is $\frac{dy}{dx}$ at this point?

10. Consider the rational function $f(x) = \frac{2x}{x+5}$.

1. Find the location and value of any relative and absolute extrema of f .
2. Describe the concavity of f throughout its domain.
3. Find any asymptotes (vertical, horizontal, or oblique/slant).
4. Sketch a careful graph of f .
5. What are the domain and range of f ?

11. Let $f(x)$ be a differentiable function whose derivative is 0 everywhere. Carefully explain how the Mean Value Theorem implies that f is a constant function. Use a clear picture as part of your explanation.