December 17, 2015

Math 103 Final Exam

No books, paper or any electronic device may be used, other than double-sided, **handwritten** note sheet in your own handwriting, at most $8.5'' \times 11''$ in size. Please turn off your cell phones.

This examination consists of 12 multiple choice questions and 3 free response questions. Please mark your answer to each problem, and show *all* your work. The correct answer to a problem with incomplete justification will earn you *only partial credit.* So will an incorrect answer together with a good explanation of what you are doing.

NAME (PRINTED):

INSTRUCTOR:

TA:

RECITATION TIME:

My signature below certifies that I have complied with the University of Pennsylvania's *code of academic integrity* in completing this examination.

_____
Your signature

DO NOT WRITE BELOW THIS LINE

Question:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Total
Points:	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	150
Score:																

1. (10 points) Functions $f(x), g(x)$ and their derivatives are given by the following table.

x	-2	-1	0	1	2	3
$f(x)$	-1	2	$1/2$	1	3	4
$g(x)$	2	1	0	-1	3	-2
$f'(x)$	1	4	2	3	-1	-2
$g'(x)$	-1	1	2	3	$1/2$	1

(a) Let $h(x) = f(g(x))$. Compute $h^{-1}(4)$.

(b) Let $h(x) = f^{-1}(x)$. Compute $h'(2)$.

(c) Let $h(x) = f(1-x) \cdot g(x+2)$. Compute $h'(1)$.

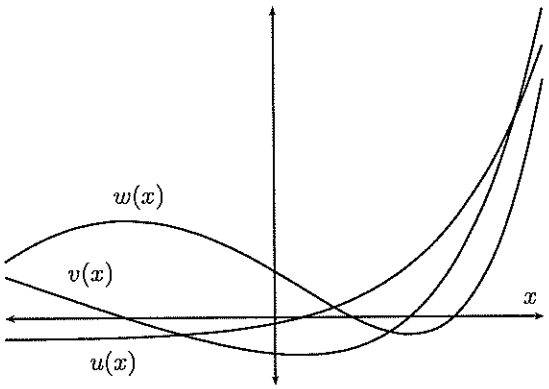
2. (10 points) For which values of the constants u and v is the following function differentiable at every value of x?

$$f(x) := \begin{cases} -1 + ux & x < 0 \\ v + e^x & x \geq 0 \end{cases}$$

(a) $u = 1$, $v = -2$ (b) $u = -1$, $v = 2$ (c) $u = -2$, $v = 1$

(d) $u = 2$, $v = -1$ (e) $u = 1$, $v = 1$ (f) none of the above

3. (10 points) To the right is a graph of $f(x)$, $f'(x)$, and $f''(x)$ for some unspecified function $f(x)$. Using the labels provided on the graph, identify the functions. Make sure to explain how you know your solution must be correct.



(a) $f = u$, $f' = v$, $f'' = w$

(b) $f = u$, $f' = w$, $f'' = v$

(c) $f = w$, $f' = u$, $f'' = v$

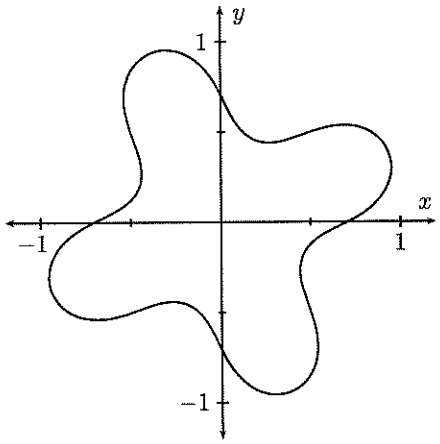
(d) $f = w$, $f' = v$, $f'' = u$

(e) $f = v$, $f' = u$, $f'' = w$

(f) $f = v$, $f' = w$, $f'' = u$

4. (10 points) To the right is a plot of the curve

$$(x^2+y^2)^4 = \frac{1}{16} + 4x^3y - 4xy^3.$$



Verify that the point $(\frac{1}{2}, \frac{1}{2})$ is on the curve and find the slope of the tangent line at that point.

(a) -1 (b) $-\frac{1}{2}$ (c) $-\frac{1}{3}$

(d) $\frac{1}{3}$ (e) $\frac{1}{2}$ (f) 1

5. (10 points) If $f(x) = x + x^3$, what is the derivative of $f^{-1}(s)$ at $s = 2$?

(a) $-\frac{1}{4}$ (b) $-\frac{1}{2}$ (c) -1 (d) 1 (e) $\frac{1}{2}$ (f) $\frac{1}{4}$

6. (10 points) Imagine you are riding a Ferris wheel of radius 10 m; the center of the wheel is 11 m above the ground. Starting at the bottom of the wheel, you ride around it counterclockwise until the moment that you are 5 m above the ground. At that instant, suppose that your height above the ground is increasing at a rate of $4\,\mathrm{m\,s^{-1}}$. What will your horizontal speed be at that same instant?

(a) $0\,\mathrm{m\,s^{-1}}$ (b) $1\,\mathrm{m\,s^{-1}}$ (c) $2\,\mathrm{m\,s^{-1}}$ (d) $3\,\mathrm{m\,s^{-1}}$ (e) $4\,\mathrm{m\,s^{-1}}$ (f) $5\,\mathrm{m\,s^{-1}}$

7. (10 points) A certain function $y(t)$ satisfies the equation

$$\frac{dy}{dt} = \frac{t}{1+t^2+y^2}.$$

Without solving this equation, determine whether or not this function has any global extrema on $(-\infty, \infty)$, and if so, state where they must occur and classify them as maxima or minima. Circle the answer below which agrees with the total number of global extrema that there must be.

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4 (f) 5

8. (10 points) Evaluate the following limit

$$\lim_{x\to 0}\left(\frac{1}{\ln(x+1)}-\frac{1}{x}\right)$$

(a) 1 (b) -1 (c) -2 (d) 2 (e) $\frac{1}{2}$ (f) $-\frac{1}{2}$ (g) 0 (h) The limit is not defined

9. (10 points) Farmer Jane needs a new rectangular pen for her pigs. She'd like it to enclose as much area as possible, but she has only 16 meters of fence to build it. She decided to build it up against the side of the barn, so that she only needs to line three sides of the pen with fence material. What's the largest possible area that she can enclose under these circumstances?

(a) $2\,\mathrm{m}^2$ (b) $4\,\mathrm{m}^2$ (c) $8\,\mathrm{m}^2$ (d) $16\,\mathrm{m}^2$ (e) $32\,\mathrm{m}^2$ (f) $64\,\mathrm{m}^2$

10. (10 points) Apply Newton's method ten times to the function $f(x) = e^{x^2}$ with initial guess $x_0 = 1$, obtaining successive approximations $x_1, x_2, \ldots, x_{10}$. Which of the following is correct?

(a) $x_{10} = 1$ (b) $x_{10} = -1$ (c) $x_{10} = -2$ (d) $x_{10} = 2$
(e) $x_{10} = \frac{1}{2}$ (f) $x_{10} = -\frac{1}{2}$ (g) $x_{10} = 0$ (h) None of the above

Do these successive values appear to converge to a solution of $f(x) = 0$?

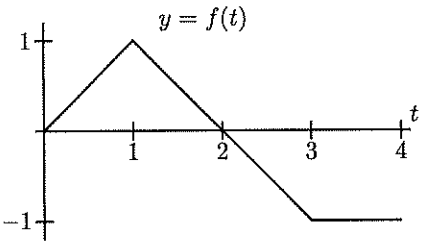
11. (10 points) The sum

$$\sum_{k=1}^{n}\left[-1+\left(\frac{2k-1}{n}\right)^2\right]\frac{2}{n}$$

is a midpoint rule Riemann sum for a certain integral. Identify the function being integrated and the interval of integration. Then determine the limit of the sum as $n \to \infty$.

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{4}{3}$ (d) $\frac{8}{3}$ (e) $\frac{16}{3}$ (f) 0 (g) $-\frac{1}{3}$ (h) $-\frac{2}{3}$

12. (10 points) Let $g(x) = \int_0^x f(t)dt$ for the function $y = f(t)$ whose graph is given by



(a) For what value of x is $g(x)$ maximized?

(b) What is $g(4)$?

(c) What is $g'(3)$?

(d) What is $g''(2)$?

13. (10 points) Compute the value of the integral

$$\int_0^{\frac{1}{2}\ln 3} \frac{e^t}{1+e^{2t}}dt.$$

(a) $\frac{\pi}{12}$ (b) $\ln 12$ (c) e^{12} (d) $\frac{e}{12}$ (e) $\frac{1}{12}$ (f) 12.

14. (10 points) What is the area of the region bounded between the curves $y = x^2 - 2x$ and $y = x - 2$?

(a) $\frac{1}{6}$ (b) 2 (c) $-\frac{2}{3}$ (d) $\frac{2}{3}$ (e) $\frac{1}{12}$ (f) $-\frac{1}{6}$ (g) $-\frac{1}{12}$ (h) -2

15. (10 points) a) Given that $\sinh(x) = \frac{3}{4}$, what is $\cosh(x)$? What is $\coth(x)$?

b) What is $\dfrac{dy}{dx}$, for $y = \ln(\cosh(5x))$?