15 pts.

1. Answer the following open ended questions:
   (a) Find all asymptotes of

$$f(x) = \frac{3x^2 - 4x + 3}{2x^2 + 5x - 12}$$

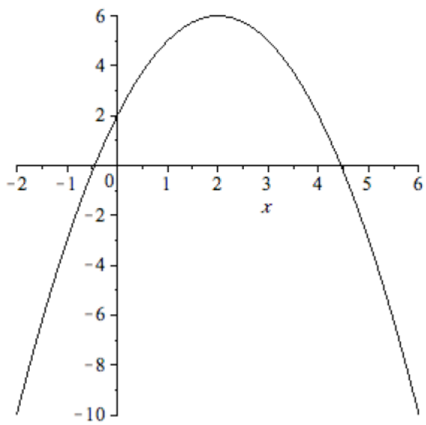
   (b) Evaluate the limit

$$\lim_{x \to 3} \frac{x^3 + 4x^2 - 21x}{x^2 - 11x - 24}$$

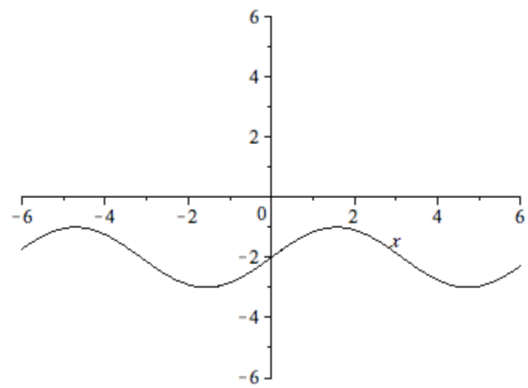
   (c) Let $f(x) = \ln(\sec x + \tan x)$. Find the derivative and simplify it.

15 pts. 2. There are two functions f and g. The graphs of their derivatives are given below.
For each given statement: Write TRUE if the statement MUST be true.
Write FALSE if the statement must be false. Write NED if there is not
enough information to decide. (e.g. if the statement might be true and might be false).

$g'(x)$



$f'(x)$



(i) $g''(x)$ is a decreasing function

(ii) $f(x)$ is a decreasing function

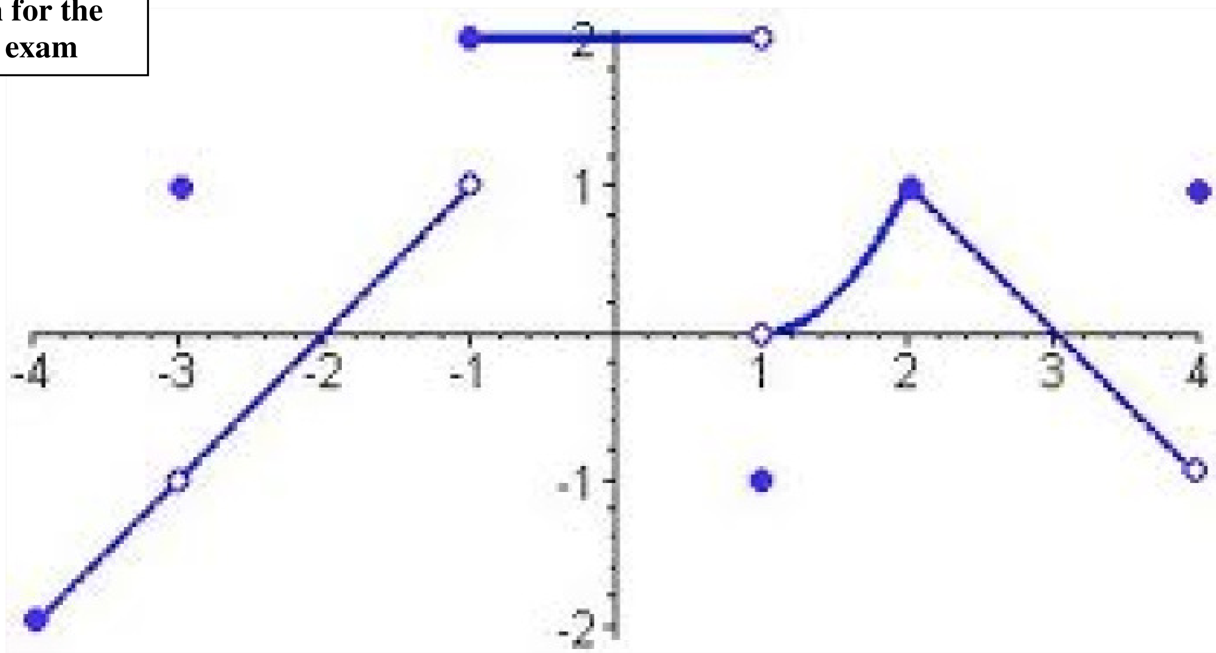
(iii) $g(x)$ is always concave down

(iv) $f(x)$ has no local minima

(v) $f(x)$ is never zero

3. For the function $y = f(x)$ whose graph is given below which ONE of the following statements is true?

5 pts. each for the rest of the exam



(a) $\lim_{x\to -3} f(x) = f(-3)$

(b) $\lim_{x\to -1} f(x) = 2$

(c) $\lim_{x\to -1^+} f(x) = 1$

(d) $\lim_{x\to -3} f(x) = -1$

(e) $\lim_{x\to 4^-} f(x) = 1$

4. The equation of the line that is tangent to the graph of $y^4 - xy^2 + x^4 = 1$ at the graph point $(1,1)$ is

(a) $3x - 2y = 5$

(b) $2x - 3y = 1$

(c) $3x + 2y = 1$

(d) $3x + 2y = 5$

(e) $3x + 2y = 0$

5. On planet Wotan , the force of gravity is such that an arrow shot vertically upward with an initial velocity of 48 meters per second will have a position function (height) $H(t) = 48t - 64t^2$. Which ONE of the following statements is true?

(a) The maximum height of the arrow is 9 meters and it hits the ground after a $\frac{3}{2}$ second flight

(b) The maximum height of the arrow is 16 meters and it hits the ground after a 2 second flight

(c) The maximum height of the arrow is 9 meters and it hits the ground after a 1 second flight

(d) The maximum height of the arrow is 9 meters and it occurs at time $t = \frac{3}{8}$

(e) The maximum height of the arrow is 16 meters and it occurs at time $t = \frac{3}{8}$

6. A plane flying horizontally at an altitude of 1 mile and a speed of 500 miles/hour passes directly over a radar station. Find the rate at which the distance from the plane to the radar station is increasing when the distance from the plane to the radar station is 2 miles.

(a) 250 mph

(b) $\dfrac{250}{\sqrt{3}}$ mph

(c) $500\sqrt{3}$ mph

(d) $50\sqrt{3}$ mph

(e) $250\sqrt{3}$ mph

7. For $y = f(x) = 4x^3 - 3x^4$, which ONE of the following statements is true?

(a) f has a local minimum at $x = 0$ and a local maximum at $x = 1$

(b) f has a local maximum at $x = 1$ and 2 points of inflection

(c) f has two points of inflection but no local maximum

(d) f has a local minimum at $x = 0$ and a local maximum at $x = 1$

(e) f has a local minimum at $x = 0$ and a point of inflection at $x = 1$

8. The slope of the line that is tangent to the graph of
$y = f(x) = x^{2x}$ at the point $(1,1)$ is

(a) -2

(b) 2

(c) $\frac{1}{2}$

(d) 1

(e) -1

9. If $y = f(x) = x^3 + 3x + 2$ and if $g(x) = f^{-1}(x)$ then
$g'(2) =$

(a) 0

(b) $\frac{1}{3}$

(c) 3

(d) $\frac{1}{16}$

(e) 16

10. Evaluate the limit below.

$$\lim_{x\to 0}\frac{x^3}{x-\tan x}$$

(a) -3

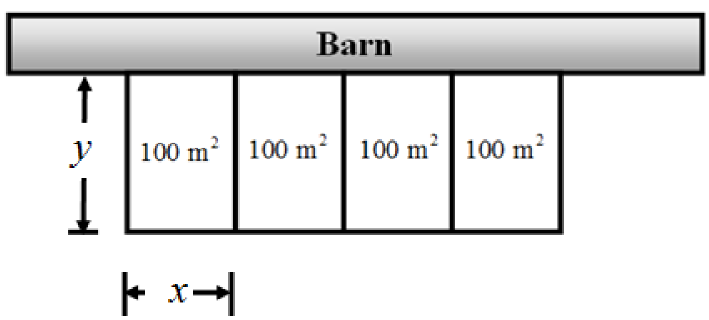
(b) -2

(c) 0

(d) 1

(e) 2

11. A rancher plans to make four identical and adjacent rectangular pens against a barn, each with an area of 100 m^2. What are the dimensions of each pen that minimize the amount of fence that must be used. (There is no fence along the barn).



(a) $x=10\text{ m},\ \ y=10\text{ m}$

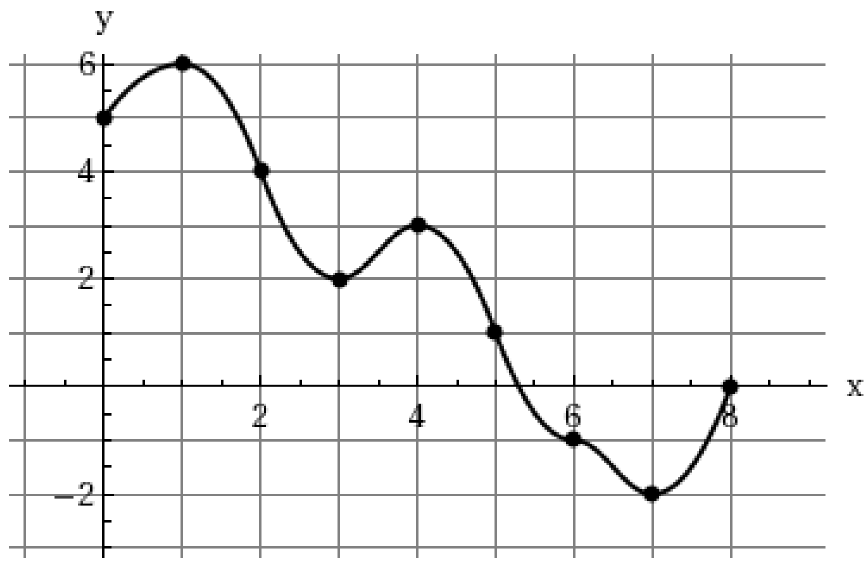
(b) $x=5\sqrt{5}\text{ m},\ \ y=4\sqrt{5}\text{ m}$

(c) $x=2\sqrt{5}\text{ m},\ \ y=10\sqrt{5}\text{ m}$

(d) $x=4\text{ m},\ \ y=25\text{ m}$

(e) $x=\dfrac{25}{3}\text{ m},\ \ y=12\text{ m}$

12. The graph of a function $f(x)$ on the interval $[0,8]$ is shown in the figure below. Estimate $\int_0^8 f(x)\,dx$ by computing the Riemann sum using four subintervals of equal length and choosing the evaluation points to be the midpoint of the subintervals.



(a) 8

(b) 10

(c) 12

(d) 14

(e) 22

13. Find the average value of

$$f(x) = xe^{x^2}$$

over the interval $[0,2]$.

Recall that the average value of $f(x)$ on the interval $[a,b]$ is found by

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

(a) 0

(b) $e^2 - 1$

(c) $e^4 - 1$

(d) $\frac{1}{2}\left(e^4 - 1\right)$

(e) $\frac{1}{4}\left(e^4 - 1\right)$

14. Let

$$G(x) = \int_{\sqrt{x}}^{2} \frac{\pi \sin\left(t^2\right)}{t}\,dt$$

Find the derivative of G evaluated at $\frac{\pi}{6}$.

(a) 1

(b) $\frac{1}{2}$

(c) $\frac{-3\sqrt{3}}{2}$

(d) $\frac{-3}{2}$

(e) 0

15. Evaluate the integral below.

$$\int_0^{12} \frac{3}{\sqrt{1+2x}}\,dx$$

(a) 6

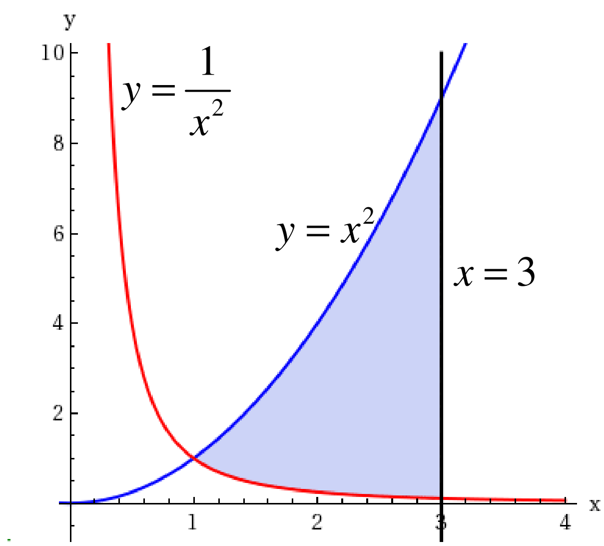
(b) 8

(c) 10

(d) 12

(e) 16

16. Find the area of the region in the first quadrant bounded by the graphs of $y = x^2$, $y = \dfrac{1}{x^2}$, and $x = 3$. See the shaded region below.



(a) 4

(b) 8

(c) 12

(d) 16

(e) 24

Problem	Answer
1	$a) y=\frac{3}{2}, x=-4, x=\frac{3}{2}$ $b)$ 0 $c) \sec x$
2	**T,T,F,T,NED**
3	**D**
4	**D**
5	**D**
6	**E**
7	**B**
8	**B**
9	**B**
10	**A**
11	**B**
12	**D**
13	**E**
14	**D**
15	**D**
16	**B**