University of Pennsylvania
Math 103
Spring 2010
Final Exam

Name _____

Recitation Day and Time _____

This exam has 14 multiple choice questions worth 10 points each and 4 open ended questions worth 15 points each. Partial credit will be given for the entire exam so be sure to show all work. On the multiple choice, write the letter choice of the correct answer in the blank space before the question number. Give supporting work, a correct answer with little or no supporting work will receive little or no credit. Use the space provided to show all work. Two sheets of scrap paper are provided at the end of the exam. If you write on the back of any page or if you want the work on the scrap paper graded please indicate this in some way.

You have 2 hours to complete the exam. You are not allowed the use of a calculator or any other electronic device. You are allowed to use the front and back of a standard 8.5"X11" sheet of paper for handwritten notes. Please silence and put away all cell phones and other electronic devices. When you finish, please stay seated until the entire time has elapsed. When time is up continue to stay seated quietly until someone comes by to collect your exam.

This table is used for grading. Please don't write in it.

Problem	Points
1	
2	
3	
4	
5	
6	
7	
8	
9	

Problem	Points
10	
11	
12	
13	
14	
15	
16	
17	
18	
Total	

Multiple Choice

____ 1. If a ball is thrown into the air with a velocity of 80 ft/s, its height in feet after t seconds is given by $s(t) = 80t - 16t^2$. It will be at maximum height when its instantaneous velocity is zero. Find its average velocity from the time it is thrown ($t = 0$) to the time it reaches its maximum height.

a. 50
b. 60
c. 100
d. 80
e. 40
f. 30
g. 48
h. 32

____ 2. Find the value of the limit $\lim_{x \to 1} \frac{2x^2 + x - 3}{x^2 - x}$.

a. 5
b. 4
c. 3
d. 2
e. $\frac{7}{2}$
f. $\frac{3}{2}$
g. $\frac{1}{2}$
h. 0

____ 3. Find the value of the limit $\lim_{x \to 1} \frac{x - 1}{\sqrt{x} - 1}$.

a. $\sqrt{2}$
b. 1
c. –1
d. $-\sqrt{2}$
e. 2
f. –4
g. 4
h. –2

____ 4. At what value(s) of x is the function $f(x) = \begin{cases} x + 2 & \text{if } x \le -1 \\ x^2 & \text{if } -1 < x < 1 \\ 3 - x & \text{if } x \ge 1 \end{cases}$ discontinuous?

a. –1
b. 0
c. 1
d. –1, 0
e. 0, 1
f. –1, 1
g. –1, 0, 1
h. None

____ 5. Find the value of the limit $\lim_{x \to \infty} \frac{4x^2 - 3x + 5}{7 + 2x - 2x^2}$.

a. $-\frac{1}{2}$
b. 1
c. 2
d. $\frac{1}{2}$
e. –1
f. $-\frac{4}{7}$
g. –2
h. $\frac{4}{7}$

____ 6. At how many different values of x does the curve $y = x^3 - 2x$ have a tangent line parallel to the line $y = x$?

a. 0
b. 1
c. 2
d. 3
e. 4
f. 5
g. 6
h. 7

____ 7. Find the slope of the tangent line to the curve $y = \frac{1}{1+2x}$ at the point $(1, \frac{1}{3})$.

a. $-\frac{1}{4}$
b. 2
c. -1
d. $\frac{1}{4}$
e. $-\frac{2}{9}$
f. -2
g. 1
h. $\frac{1}{2}$

____ 8. If $f(x) = \sqrt[3]{x^2 - 1}$, find $f''(3)$.

a. $\frac{1}{4}$
b. $\frac{1}{12}$
c. $-\frac{1}{12}$
d. $\frac{3}{4}$
e. $\frac{1}{3}$
f. $\frac{4}{3}$
g. $-\frac{4}{3}$
h. None of these

____ 9. Find the difference between the local maximum and the local minimum values of the function $f(x) = x^3 - 3x + 27$.

a. 4
b. 1
c. 9
d. 2
e. 6
f. 5
g. 8
h. 3

____ 10. Find the absolute minimum value for the function $f(x) = 2 \sin x \cos x$ for $0 \le x \le \pi$.

a. 1
b. -2
c. $\frac{1}{4}$
d. 0
e. -1
f. 2
g. $\frac{1}{2}$
h. No absolute minimum

____ 11. Find the interval on which $f(x) = xe^{-x}$ is increasing.

a. $(-\infty, 1)$
b. $(-\infty, 2)$
c. $(1, \infty)$
d. $(1, 2)$
e. $(1, e)$
f. $(-\infty, e)$
g. $(-\infty, \infty)$
h. $\left(-\infty, \frac{1}{e}\right)$

____ 12. Find the x-coordinate of the point of inflection of the function $f(x) = x^3 - x^2 - x + 1$.

a. $-\frac{3}{4}$
b. $-\frac{3}{2}$
c. $\frac{3}{4}$
d. $-\frac{1}{3}$
e. $\frac{3}{2}$
f. $\frac{1}{3}$
g. $-\frac{3}{4}$
h. $\frac{2}{3}$

____ 13. Find the value of the limit $\lim_{x \to 0^+} \frac{x}{\sin x + \tan x}$.

a. -2
b. 0
c. $-\frac{1}{2}$
d. $\frac{1}{4}$
e. 2
f. ∞
g. $\frac{1}{2}$
h. 4

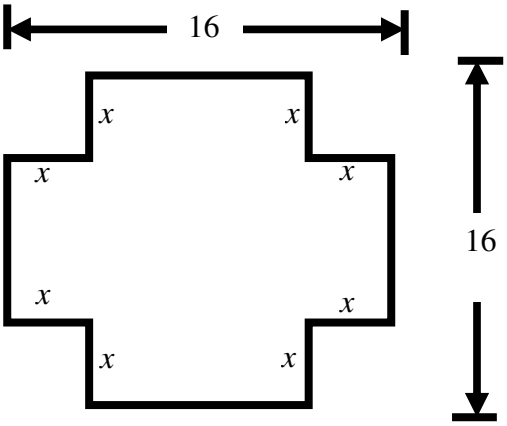
____ 14. Find the value of the integral $\int_1^2 \frac{x^2 - 1}{x}\, dx$.

a. $\frac{1}{2}$
b. 1
c. $\frac{3}{2}$
d. 2
e. $\frac{1}{2}\ln 2$
f. $1 + \ln 2$
g. $\frac{3}{2} - \ln 2$
h. $2 + \ln 2$

Open Ended

15. Suppose that f is a continuous function and $f(0) = 0,\ f(2) = f(-2) = 1,\ f'(0) = 0,\ f'(x) > 0$ for $x > 0, f'(x) < 0$ for $x < 0, f''(x) > 0$ for $|x| < 2, f''(x) < 0$ for $|x| > 2$, and $\lim_{x \to \infty} f(x) = \lim_{x \to -\infty} f(x) = 2$. Sketch the graph of $y = f(x)$.

16. An open box is made from a 16 inch × 16 inch piece of cardboard by cutting equal squares from each corner and folding up the sides. For maximum volume, what size squares should be cut out?



17. The base of a right triangle is increasing at a rate of 1 cm./sec. The height of the triangle stays constant at 4 cm. At what rate is the angle opposite the base changing when the triangle is isosceles?

18. Evaluate

a) $\int_0^1 \frac{x^2}{\left(1+x^3\right)^2}\,dx$

b) $\int_0^{\pi/2} \sqrt{\cos x}\,\sin x\,dx$

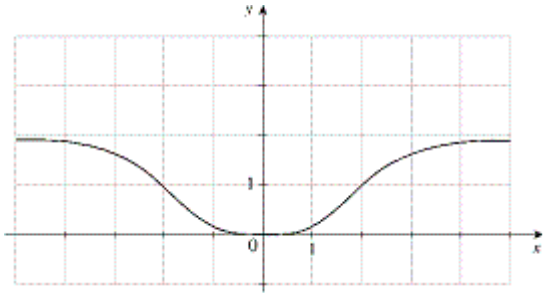
Math 103 Spring 2010 Final Exam
Answer Section

MULTIPLE CHOICE

1. E
2. A
3. E
4. C
5. G
6. C
7. E
8. C
9. A
10. E
11. A
12. F
13. G
14. G

Open Ended

15.



16. $\frac{8}{3} \times \frac{8}{3}$ in. squares

17. 1/8 rad./sec.

18. a) 1/6 b) 2/3