Math 103 — Final Exam. NAME: ____________________________

Remember to write all your final answers on the answer sheet.

1. Which of the following is true for **ALL** real numbers a, b, and c?
   (A) If $a \leq b$, then $ac \leq bc$.
   (B) If $a < b < 0$, then $1/a < 1/b < 0$.
   (C) $|-a| = a$.
   (D) $|a+b| \geq |a| + |b|$.
   (E) $\sqrt[3]{a^3} = |a|$.
   (F) If $a^2 = b^2$, then $a = b$.
   (G) all of the above
   (H) none of the above

2. Find the linearization of $f(x) = 1/x$ at the point $a = 1/2$.
   (A) $L(x) = -4x + 2$
   (B) $L(x) = -4x + 4$
   (C) $L(x) = -2x + 2$
   (D) $L(x) = -2x + 3$
   (E) $L(x) = -(1/2)x + 2$
   (F) $L(x) = -(1/2)x + 9/4$
   (G) $L(x) = -(1/4)x + 2$
   (H) $L(x) = -(1/4)x + 9/8$

3. Find the average value of the function $f(x) = x^2 + 1$ on the interval $[-1, 1]$.
   (A) 0
   (B) 1/3
   (C) 1
   (D) 4/3
   (E) 5/3
   (F) 2
   (G) 8/3
   (H) 10/3

4. Let $f(x) = e^{2x} + \ln((x+1)^3)$. Compute $f'(0)$.
   (A) -1
   (B) 0
   (C) 1
   (D) 2
   (E) 3
   (F) 4
   (G) 5
   (H) 6

5. Compute the indefinite integral

$$\int \sin^2 \theta \, d\theta.$$

(A) $\frac{1}{3}\sin^3\theta + C$
(B) $-\cos^2\theta + C$
(C) $(1 - \cos 2\theta)/2 + C$
(D) $(1 + \cos 2\theta)/2 + C$
(E) $\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta + C$
(F) $\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta + C$
(G) $\frac{1}{2}\theta + \frac{1}{2}\sin 2\theta + C$
(H) $\frac{1}{2}\theta - \frac{1}{2}\sin 2\theta + C$

6. Consider the following six functions:

$$\begin{array}{lll} f(x) = \tan x, & g(x) = 1/x, & h(x) = e^x, \\ u(x) = |x|, & v(x) = \sqrt[3]{x}, & w(x) = \ln x. \end{array}$$

How many of these functions are continuous at $x = 0$?
How many of these functions are differentiable at $x = 0$?
(A) 3 functions are continuous at zero, 1 function is differentiable at zero.
(B) 3 functions are continuous at zero, 2 functions are differentiable at zero.
(C) 4 functions are continuous at zero, 2 functions are differentiable at zero.
(D) 4 functions are continuous at zero, 3 functions are differentiable at zero.
(E) 5 functions are continuous at zero, 3 functions are differentiable at zero.
(F) 5 functions are continuous at zero, 4 functions are differentiable at zero.
(G) 6 functions are continuous at zero, 4 functions are differentiable at zero.
(H) 6 functions are continuous at zero, 5 functions are differentiable at zero.

7. Find the area of the region bounded by the graphs of $y = e^x$, $y = x$, $x = 0$, and $x = 2$.
(Note: The graph of $y = x$ is below the graph of $y = e^x$ for all x.)
(A) $e^2 - 5$
(B) $e^2 - 4$
(C) $e^2 - 3$
(D) $e^2 - 2$
(E) $e^2 - 1$
(F) e^2
(G) $e^2 + 1$
(H) $e^2 + 2$

8. Let $g(x) = x^3 + bx^2 + 5x - 7$, where b is some constant. Find the value of b such that g has an inflection point at $x = -1$.
(A) -4
(B) -3
(C) -2
(D) -1
(E) 0
(F) 1
(G) 2
(H) 3

9. We have
$$\ln 4 = \int_1^4 \frac{1}{x}\, dx.$$
Using the trapezoid method with $n = 3$ intervals, give an approximation for $\ln 4$.
(A) 3/2
(B) 7/5
(C) 11/6
(D) 13/12
(E) 25/12
(F) 25/24
(G) 35/24
(H) 25/36

10. Find the equations of all the asymptotes of
$$y = f(x) = \frac{2x^3 + 3x^2 + 1}{x^2 - x},$$
including any oblique asymptotes.
(A) $x = 0,\ x = 1.$
(B) $x = 0,\ x = -1.$
(C) $x = 0,\ x = 1,\ y = 2.$
(D) $x = 0,\ x = -1,\ y = 2.$
(E) $x = 0,\ x = 1,\ y = 2x + 1.$
(F) $x = 0,\ x = -1,\ y = 2x + 1.$
(G) $x = 0,\ x = 1,\ y = 2x + 5.$
(H) $x = 0,\ x = -1,\ y = 2x + 5.$

11. What is the output of the following Maple statement? [Draw a picture.]

```
> int(sqrt(9-x^2),x=-3..3);
```

(A) 3π
(B) $9\pi/4$
(C) $9\pi/2$
(D) 9π
(E) 0
(F) 3
(G) 6
(H) none of the above

12. Suppose g is a continuous differentiable function defined on $\mathbb{R}$ such that:

$$\begin{array}{lll} g(1) = 2, & g'(1) = 0, & g''(1) = -3, \\ g(4) = -1, & g'(4) = 0, & g''(4) = 0. \end{array}$$

Which of the following statements **MUST** be true?

I. g has a local maximum at $x = 1$.
II. g has a local minimum at $x = 4$.
III. $g'(c) = -1$ for some c between 1 and 4.

(A) I only
(B) II only
(C) III only
(D) I and II only
(E) I and III only
(F) II and III only
(G) I, II, and III
(H) none of the statements I, II, III is necessarily true

---

13. Let f be the function with domain $[0, 3]$ defined by $f(x) = \sqrt{x+1}$ for $0 \le x \le 3$.

(a) Find the absolute minimum value of f, the absolute maximum value of f, and the range of f.

(b) Give a formula for the inverse function for f.
State the domain and range of this inverse function.

(c) Find the volume of the solid obtained by rotating the graph of f around the x-axis.

(d) Write down a definite integral giving the volume of the solid obtained by rotating the graph of f around the y-axis. Do not compute the value of this integral.

SHOW YOUR WORK ON THE ANSWER SHEET.

---

14. Suppose g is a continuous function defined on $\mathbb{R}$, g is an *even* function, and we know that

$$\int_1^2 g(x)\,dx = 3 \quad \text{and} \quad \int_1^4 g(x)\,dx = 7.$$

Compute each of the following integrals, *or state that there is not enough information to answer.*

(a) $\displaystyle\int_2^4 g(x)\,dx.$

(b) $\displaystyle\int_4^1 g(x)\,dx.$

(c) $\displaystyle\int_1^2 5g(s)\,ds.$

(d) $\displaystyle\int_{-2}^{-1} g(x)\,dx.$

(e) $\displaystyle\int_1^2 tg(t^2)\,dt.$

(f) $\displaystyle\int g(x)\,dx.$

SHOW YOUR WORK ON THE ANSWER SHEET.

---

15. A right triangle is inscribed in the region above the x-axis and below the parabola $y = 9 - x^2$, with one leg on the x-axis and the other leg parallel to the y-axis. What is the largest area such a triangle can have? What are the base and height of the triangle with this area? Also, explain how you know that the triangle with these dimensions really does give the absolute maximum area.

SHOW YOUR WORK ON THE ANSWER SHEET.

---

16. Compute the following limits, showing all work. If a limit does not exist, explain why.

(a) $\displaystyle\lim_{x\to 5}(3x+4).$

(b) $\displaystyle\lim_{x\to\pi^+} \cot x.$

(c) $\displaystyle\lim_{x\to-\infty} \frac{-x^3+x}{2x^4+3x^2}.$

(d) $\displaystyle\lim_{x\to 0}\left(\frac{d}{dx}|x|\right).$

BONUS:

$$\lim_{h\to 0}\left[\frac{\displaystyle\int_0^{\sqrt{\pi/6}+h} \sin(t^2)\,dt - \int_0^{\sqrt{\pi/6}} \sin(t^2)\,dt}{h}\right].$$

SHOW YOUR WORK ON THE ANSWER SHEET.

— **END OF EXAM** —