1. Suppose that f and g are integrable and that

$$\int_1^2 f(x)\,dx = -4, \quad \int_1^5 f(x)\,dx = 6, \quad \int_1^5 g(x)\,dx = 8.$$

Find

I)

$$\int_5^1 g(x)\,dx$$

II)

$$\int_2^5 f(x)\,dx$$

III)

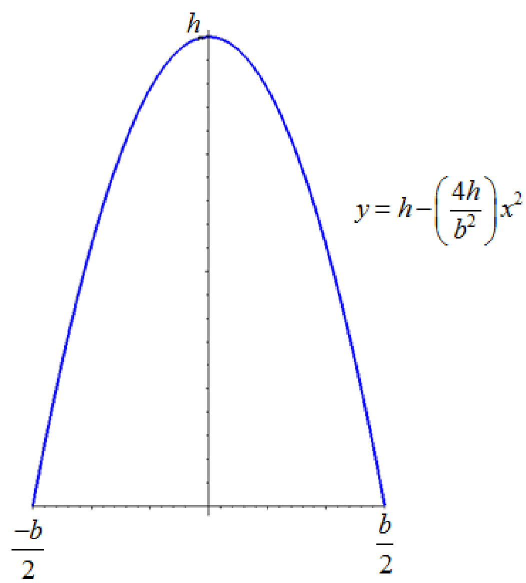
$$\int_1^5 \left[4f(x) - g(x)\right]dx$$

2. Archimedes (287-212 B.C.), inventor, military engineer, physicist, and the greatest mathematician of classical times in the Western world, discovered that area under a parabolic arch is two-thirds the base times the height. Let $h =$ height and $b =$ base

Use calculus to find the area and verify Archimedes' discovery for the parabola

$$y = h - \left(\frac{4h}{b^2}\right)x^2$$

whose graph is given to the right.
given that $h = 8$ and $b = 3$.



3. Let

$$y = \int_0^{e^{x^2}} \frac{1}{\sqrt{t}}\,dt$$

Find $y'(2)$.

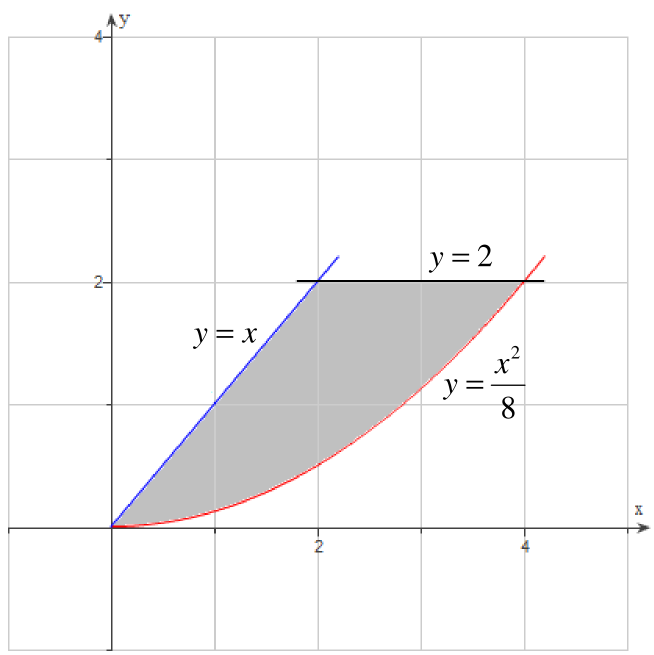
A) e
B) e^2
C) e^4
D) $2e$
E) $2e^2$
F) $4e^2$
G) 2
H) 4

4. Evaluate

$$\int_0^4 \frac{6x}{\sqrt{x^2+9}}\,dx$$

A) $\frac{1}{2}$
B) 1
C) 2
D) 4
E) 6
F) 8
G) 12
H) 16

5. Find the area of the shaded region



A) $\frac{13}{4}$
B) $\frac{10}{3}$
C) $\frac{16}{5}$
D) $\frac{8}{3}$
E) $\frac{11}{4}$
F) $\frac{14}{5}$
G) 3
H) $\frac{19}{6}$

6. For what values of a, m and b does the function

$$f(x)=\begin{cases} 3, & x=0 \\ -x^2+3x+a, & 0<x<1 \\ mx+b, & 1\le x\le 2 \end{cases}$$

Satisfy the hypotheses of the Mean Value Theorem on the interval $[0,2]$?

7. Suppose that $f'(x)=2x$ for all x and that $f(-2)=3$.

Find $f(2)$.

A) -2
B) -1
C) 0
D) 1
E) 2
F) 3
G) 4
H) 5

8. Let

$$f(x)=\frac{x^2-3}{x-2}.$$

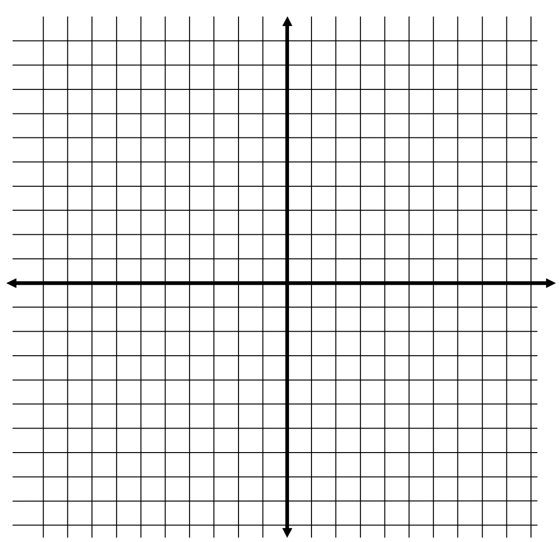
I) Find the interval(s) where the functions is increasing and where the function is decreasing.

II) Find the critical points of $f(x)$, if any, identify whether these lead to local maximum values, local minimum values, or neither.

III) Find the interval(s) where the function is concave up and where the function is concave down.

IV) Find the inflection point(s) of $f(x)$, if any.

V) Sketch the graph of $f(x)$. Take into account the domain, symmetry, intercepts, asymptotes.



9. Evaluate the limit

$$\lim_{x\to 0}\frac{e^{x}-e^{-x}-2x}{x-\sin(x)}$$

A) $\frac{1}{2}$

B) $\frac{3}{4}$

C) $\frac{1}{4}$

D) $\frac{2}{3}$

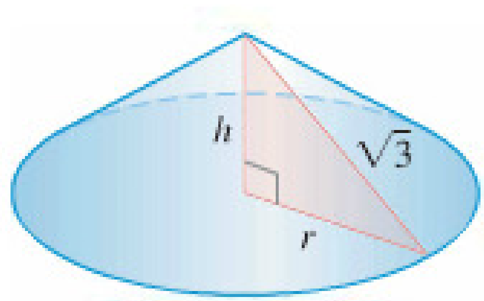
E) 0

F) 1

G) 2

H) 3

10. A right triangle whose hypotenuse is $\sqrt{3}$ m long is revolved about one of its legs to generate a right circular cone. Find the radius, height, and volume of the cone of greatest volume that can be made this way.



11. Let

$$y=(1+\cos 2t)^{-4}$$

Find $y'\left(\frac{\pi}{4}\right)$.

A) 4

B) $\frac{1}{3}$

C) $\frac{2}{3}$

D) $\frac{1}{2}$

E) 8

F) 2

G) $\frac{1}{4}$

H) $\frac{1}{8}$

12. Find the equation of the tangent line to the curve

$$6x^2+3xy+2y^2+17y-6=0$$

at $(-1,0)$.

A) $y=\frac{1}{7}x+\frac{1}{7}$

B) $y=\frac{3}{7}x+\frac{3}{7}$

C) $y=\frac{5}{7}x+\frac{5}{7}$

D) $y=\frac{-1}{7}x-\frac{1}{7}$

E) $y=\frac{2}{7}x+\frac{2}{7}$

F) $y=\frac{4}{7}x+\frac{4}{7}$

G) $y=\frac{6}{7}x+\frac{6}{7}$

H) $y=\frac{-3}{7}x-\frac{3}{7}$

13. Let

$$y=\frac{\ln x}{x}$$

Find $y'\left(\sqrt{e}\right)$.

A) $\frac{1}{e^2}$

B) $\frac{1}{2e}$

C) $\frac{e}{2}$

D) $\frac{1}{2}$

E) $\frac{e}{8}$

F) $\frac{\sqrt{e}}{4}$

G) $\frac{2}{\sqrt{e}}$

H) 0

14. Let

$$y=\ln\left(\arctan x\right)$$

Find $y'(1)$.

A) $\frac{1}{2}$

B) $\frac{\sqrt{3}}{2}$

C) π

D) $\frac{\pi}{2}$

E) 1

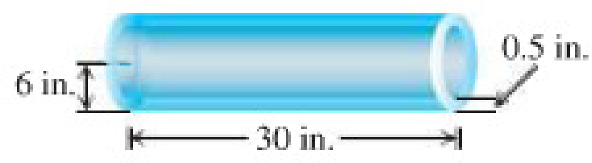
F) $\frac{4}{\pi}$

G) $\frac{3}{\pi}$

H) $\frac{2}{\pi}$

15. Charlotte flies a kite at a height of 300 ft, the wind carries the kite horizontally away from her at a rate of 25 ft./sec. How fast must she let out the string when the length of the string is 500 ft.?

16. Estimate the volume of material in a cylindrical shell with length 30 in., radius 6in., and shell thickness 0.5 in. using differentials.



17. Find the slope of the tangent line to the function $y = 5 - x^2$ at the point $(1,4)$ **using the definition of the derivative**.

18. Let $f(x) = \sqrt{19 - x},\ x_0 = 10,\ \text{and } \varepsilon = 1$. Find $L = \lim_{x \to x_0} f(x)$. Then find a number $\delta > 0$ such that for all x

$$0 < |x - x_0| < \delta \quad \Rightarrow \quad |f(x) - L| < \varepsilon$$

19. For what value of a is

$$f(x) = \begin{cases} x^2 - 1, & x < 3 \\ 2ax, & x \ge 3 \end{cases}$$

continuous at every x?

20. Evaluate

$$\lim_{x \to \infty} \left(\sqrt{x^2 + 3x} - \sqrt{x^2 - 2x} \right)$$

Answers:

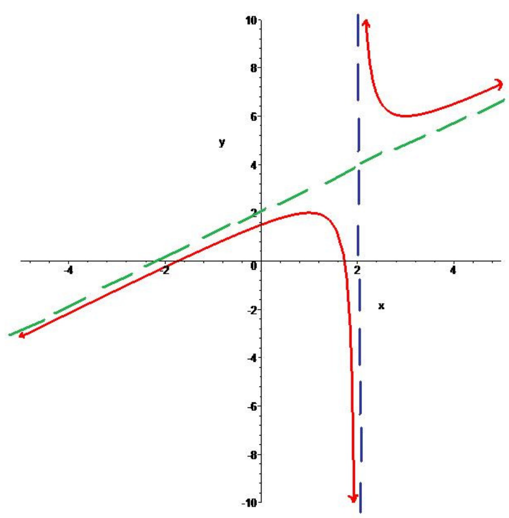
1. I) -8 II) 10 III) 16
2. 16
3. F
4. G
5. B
6. a = 3, m = 1, b = 4
7. F
8. I) Increasing: $(-\infty,1)\cup(3,\infty)$ Decreasing: $(1,3)$

   II) Critical points: $x=1$ leads to a local maximum value, $x=3$ leads to a local minimum value

   III) Concave Up: $(2,\infty)$ Concave Down: $(-\infty,2)$

   IV) No inflection points

   V)



Domain: $(-\infty,2)\cup(2,\infty)$, No Symmetry,

Vertical asymptote $x=2$, Slant asymptote $y=x+2$

$x-$int. $(\sqrt{3},0),(-\sqrt{3},0)$, $y-$int. $\left(0,\frac{3}{2}\right)$

9. G
10. $h=1,\ r=\sqrt{2},\ V=\frac{2\pi}{3}$
11. E
12. G
13. B
14. H
15. 20 ft/s
16. $180\pi\ \text{in}^3$
17. $m=-2$
18. $L=3,\ \delta=5$
19. $a=\frac{4}{3}$
20. $\frac{5}{2}$