1. Compute

$$\lim_{t\to 0}\frac{t^2}{\ln(\sec t)}$$

A) $-\infty$ B) -2 C) -1 D) 0 E) 1 F) 2 G) ∞ H) None of these

2. Given

$$\lim_{y\to 3}\frac{f(y)-5}{y-3}=5.$$

What is $\lim_{y\to 3} f(y)$?

A) $-\infty$ B) 0 C) 1 D) 2 E) 3 F) 5 G) ∞ H) Cannot be determined

3. Compare the values

$$L=\lim_{x\to\infty}\frac{5+4x-8x^2}{-4x^2+3x+1}\ , \qquad M=\lim_{x\to 0}\frac{\int_0^x e^{-t^2}\,dt}{x}\ , \qquad N=\lim_{x\to 1^+} x^{\frac{1}{x-1}}\ .$$

A) $N=M, L$ does not exist
B) $N<L<M$
C) $L<N, M$ does not exist
D) $L<N<M$
E) $N=M<L$
F) $M<L<N$
G) $M<N<L$
H) $L<M, N$ does not exist

4. A taxi drove 80 miles in 2 hours. Show that at least twice that the taxi was running at exactly 30 miles per hour (assuming the initial and final speeds are zero).

5. For what x value(s) is the following function **NOT** continuous?

$$f(x) = \begin{cases} 0 & \text{if } x \le -1 \\ \ln(1+x) & \text{if } -1 < x \le 0 \\ x^2 \cos\left(\frac{1}{x}\right) & \text{if } x > 0 \end{cases}$$

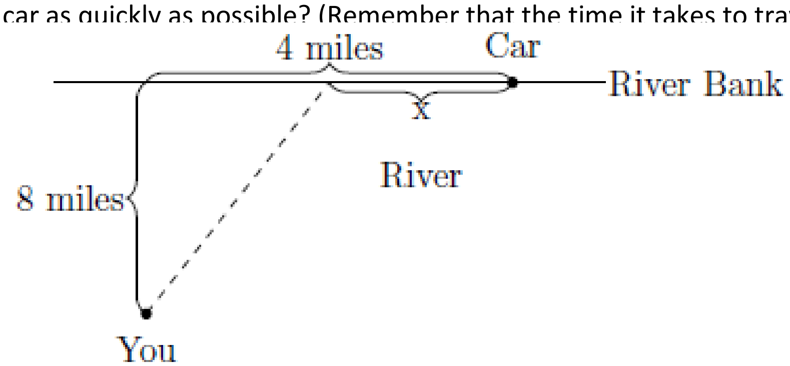
A) -1
B) 0
C) 1
D) -1 and 0
E) 0 and 1
F) -1 and 1
G) $-1, 0,$ and 1
H) f is continuous everywhere

6. You have fallen off a boat into a river. Your car, with a change of clothes, is parked on the side of the river. You would like to get there as quickly as possible so you can change into dry clothes.

The river bank is 8 miles away from you at the nearest spot, and your car is 4 miles away from that down the river bank.

You can swim at a rate of 1 mile/hour, and you can walk on land at a rate of $\sqrt{17}$ miles/hour. Your fastest route is to swim to some point on the edge of the river bank and then walk the rest of the way.

How far away from the car (the value of x in the picture below) should you reach the river bank so that you can get to the car as quickly as possible? (Remember that the time it takes to travel a distance is $\dfrac{\text{distance}}{\text{speed}}$).



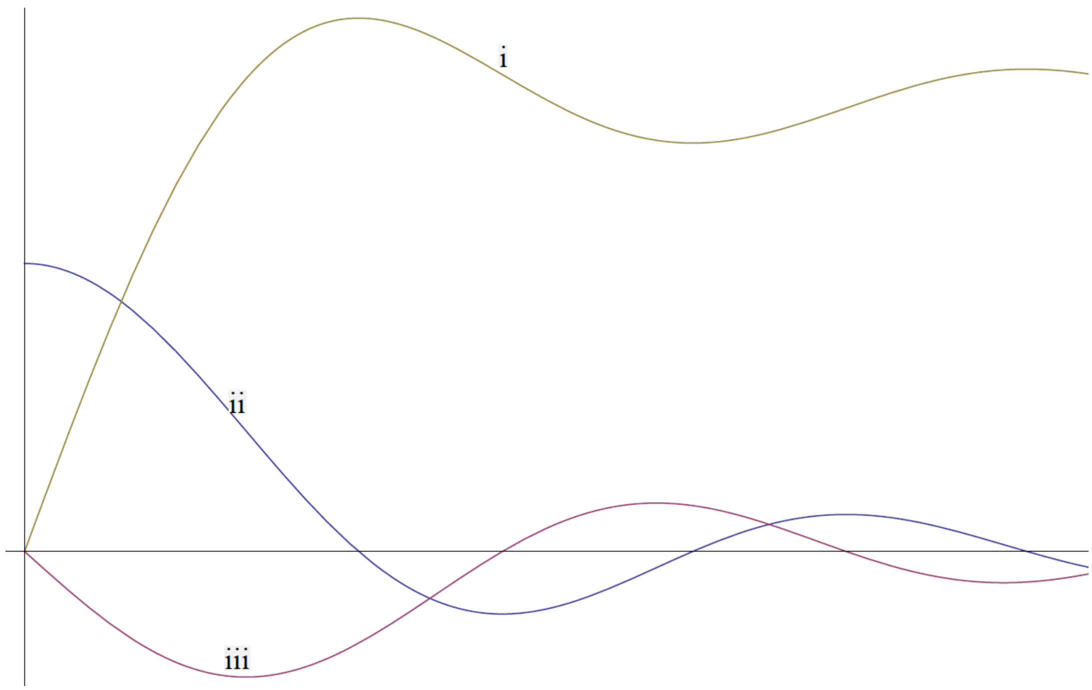
7. Some values of the functions f and g are given in the table below.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	1	2	-1	3
2	-3	-1	2	-2

If $h(x)=f(1-x)\,g(1+x)$ what is $h'(1)$?

A) -6 B) -3 C) -2 D) -1 E) 0 F) 1 G) 2 H) 6

8. The curves (i), (ii), and (iii) in the graph below are the graphs of a function f and its first and second derivatives. Which curve is f, which is f', and which is f''? Explain.



A) (i) f (ii) f' (iii) f''
B) (i) f (ii) f'' (iii) f'
C) (i) f' (ii) f (iii) f''
D) (i) f' (ii) f'' (iii) f
E) (i) f'' (ii) f (iii) f'
F) (i) f'' (ii) f' (iii) f

9. Let

$$f(x) = \sin^{-1}(2e^x)$$

What is $f'\left(\ln\left(\frac{3}{10}\right)\right)$?

A) 0 B) $\frac{3}{16}$ C) $\frac{1}{4}$ D) $\frac{1}{2}$ E) $\frac{3}{4}$ F) $\frac{13}{16}$ G) 1 H) $\frac{1}{e}$

10. A curve is given implicitly by the equation

$$x^2 - y^2 = x^2 y \ .$$

What is the tangent line to the curve at the point $\left(\frac{\sqrt{2}}{2}, \frac{1}{2}\right)$?

A) $y - \frac{1}{2} = \frac{\sqrt{2}}{2}\left(x - \frac{\sqrt{2}}{2}\right)$

B) $y - \frac{1}{2} = \frac{\sqrt{2}}{3}\left(x - \frac{\sqrt{2}}{2}\right)$

C) $y - \frac{1}{2} = \sqrt{2}\left(x - \frac{\sqrt{2}}{2}\right)$

D) $y - \frac{1}{2} = 2\left(x - \frac{\sqrt{2}}{2}\right)$

E) $y - \frac{\sqrt{2}}{2} = \sqrt{2}\left(x - \frac{1}{2}\right)$

F) $y - \frac{\sqrt{2}}{2} = 2\left(x - \frac{1}{2}\right)$

G) $y - \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}\left(x - \frac{1}{2}\right)$

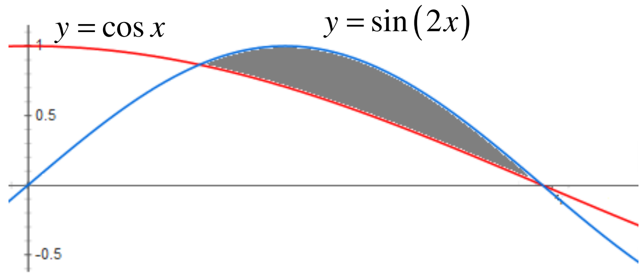
H) $y - \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{3}\left(x - \frac{1}{2}\right)$

11. Evaluate

$$\int_1^{e^3} \frac{\ln\sqrt{x}}{x}\,dx$$

A) $\frac{1}{2}$ B) $\frac{3}{4}$ C) $\frac{5}{4}$ D) $\frac{3}{2}$ E) $\frac{9}{4}$ F) $\frac{7}{2}$ G) $\frac{9}{2}$ H) $\frac{11}{2}$

12. Find the area of the shaded region.
Hint: $\sin 2x = 2\sin x\cos x$



A) $\frac{1}{6}$ B) $\frac{1}{5}$ C) $\frac{1}{4}$ D) $\frac{1}{3}$ E) $\frac{1}{2}$ F) $\frac{3}{5}$ G) $\frac{3}{4}$ H) 1

13. Evaluate

$$\frac{1}{2^9}\int_0^{64}\left(4\sqrt[3]{x}-\sqrt{x}\right)dx$$

A) $\frac{1}{6}$ B) $\frac{1}{4}$ C) $\frac{2}{5}$ D) $\frac{1}{3}$ E) $\frac{1}{2}$ F) $\frac{3}{4}$ G) $\frac{5}{6}$ H) 1

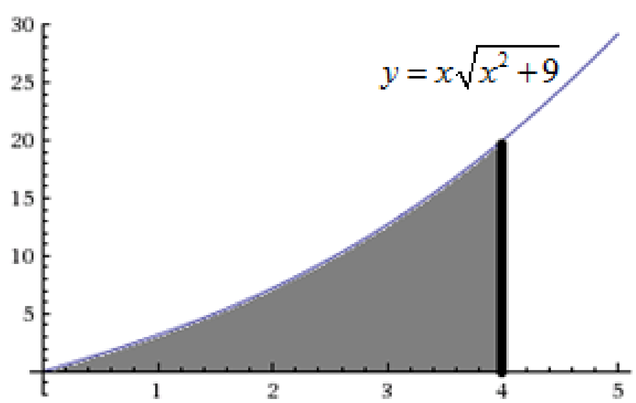
14. Let

$$f(x)=\begin{cases}12-x^2 & \text{for } x\le 2\\ x^3 & \text{for } x>2\end{cases}$$

Calculate $\int_{-1}^{4} f(x)\,dx$

a) 144 b) 126 c) 93 d) 63 e) 30 f) 20 g) 5 h) 2

15. Find the area of the shaded region.



A) $\frac{62}{3}$ B) $\frac{68}{3}$ C) $\frac{71}{3}$ D) $\frac{74}{3}$
E) $\frac{82}{3}$ F) $\frac{86}{3}$ G) $\frac{95}{3}$ H) $\frac{98}{3}$

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1. F
2. F
3. F
4. Argument based on the Intermediate Value Theorem
5. A
6. x = 2 miles
7. A
8. A
9. E
10. B
11. E
12. C
13. G
14. C
15. H