University of Pennsylvania **Math 103 Final Exam Fall 2010**
Department of Mathematics **Blair, Lieberman, and Rimmer**

Part I: 5 point Multiple Choice Questions (Numbers 1 – 6)

1. Find the equation of the tangent line to the curve $ye^x + x\ln y = e$ at the point $(1,1)$.

A) $y = \frac{-1}{e+1}(ex - 2e - 1)$ B) $y = -\frac{1}{2}(x+3)$ C) $y = 1$ D) $y = \frac{1}{2}(x+1)$

E) $y = \frac{-1}{e+1}(ex+1)$ F) $y = 2x+2$ G) $y = \frac{1}{e}x + e$ H) None of these

2. Let

$$f(x) = \begin{cases} \dfrac{x}{x^2+1} & x \le 1 \\ ax^2 + bx + 1 & x > 1 \end{cases}.$$

Find the values of a and b so that f is differentiable everywhere.

A) $a = -\frac{1}{2}$ and $b = 0$ B) $a = \frac{1}{2}$ and $b = -1$ C) $a = -1$ and $b = \frac{1}{2}$ D) $a = 1$ and $b = -\frac{3}{2}$

E) $a = 1$ and $b = 0$ F) $a = -1$ and $b = -\frac{1}{2}$ G) $a = -\frac{3}{2}$ and $b = 0$ H) No such values

3. Values of functions f, g, f', and g' are given in the table below:

x	-1	0	1	2
f	11	7	5	5
g	-3	2	-1	1
f'	1	3	4	7
g'	2	1	5	2

Let $h(x) = f(g(x)) \cdot g(x)$. What is $h'(1)$?

A) –5 B) 0 C) 1 D) 25 E) 40 F) 50 G) 100 H) 250

4. Let $f(x) = \arccos\left(\frac{1}{\sqrt{x}}\right)$. What is $f'(4)$?

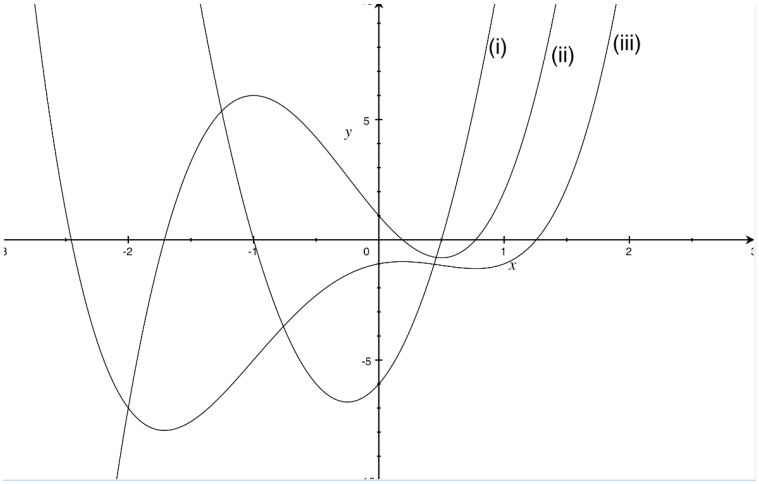
A) $-\frac{\pi}{3}$ B) $-\frac{1}{8\sqrt{3}}$ C) $\frac{2}{\sqrt{3}}$ D) $-\frac{2}{\sqrt{3}}$ E) $\frac{1}{8\sqrt{3}}$ F) $\frac{\pi}{3}$ G) $\frac{\pi}{2}$ H) $\frac{\pi}{6}$

5. Which of the following **MUST** contain a zero of the function $f(x) = \frac{1}{4}x^3 - x^2 + x - 1$?

A) $(-1,0]$ B) $(0,1)$ C) $(1,2]$ D) $(2,3)$

E) $(3,4)$ F) $(4,5)$ G) $(-2,-1)$ H) $(-3,-2)$

6. The curves (i), (ii), and (iii) in the graph below are the graphs of a function f and its first and second derivatives. Which curve is f, which is f', and which is f''? Explain.



A) $(i)\ f \quad (ii)\ f' \quad (iii)\ f''$
B) $(i)\ f \quad (ii)\ f'' \quad (iii)\ f'$
C) $(i)\ f' \quad (ii)\ f \quad (iii)\ f''$
D) $(i)\ f' \quad (ii)\ f'' \quad (iii)\ f$
E) $(i)\ f'' \quad (ii)\ f \quad (iii)\ f'$
F) $(i)\ f'' \quad (ii)\ f' \quad (iii)\ f$

Part II: 10 point Multiple Choice Questions (Numbers 7 – 10)

7. Compare the values

$$L = \lim_{x\to 0^+} \frac{\sin x - x}{x^3} \qquad M = \lim_{x\to\infty} x^{\frac{1}{\sqrt{x}}} \qquad N = \lim_{x\to 2} \frac{x^2 - 3x + 2}{4 - x^2}$$

A) $N < M$, L does not exist
B) $N < L < M$
C) $N < L$, M does not exist
D) $L < N < M$
E) $L < N = M$
F) $M < L < N$
G) $M < N < L$
H) $M < L$, N does not exist

8. Evaluate

$$\int_0^{\sqrt{3}} \frac{x}{\sqrt{x^2+1}}\,dx$$

A) -4 B) 0 C) 1 D) 2 E) 3 F) 4 G) 5 H) $\ln 3$

9. Evaluate

$$\int_0^{\frac{1}{2}} \frac{dx}{1+4x^2}$$

A) $\frac{\pi}{8}$ B) $\frac{\pi}{6}$ C) $\frac{\pi}{4}$ D) $\frac{\pi}{3}$ E) $\frac{\pi}{2}$ F) $\frac{2\pi}{3}$ G) π H) 0

10. Evaluate

$$\int_1^e \frac{\ln x}{x} e^{(\ln x)^2} dx$$

A) 0 B) $\frac{1}{2}e$ C) e^2 D) 1 E) $\frac{1}{2}(e-1)$ F) $1-e$ G) π H) e

Part III: 10 point Open-Ended Questions (Numbers 11 – 13)

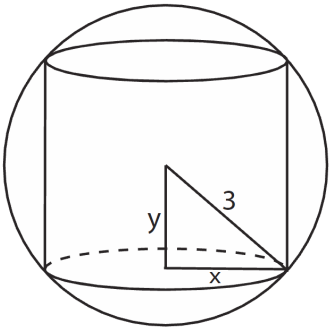
11. Give the function $f(x)=\frac{x^2+x}{x-1}$ compute the following:

1 pt. each for A and B
2 pts. each for C - F

A) Find the x-intercepts and y-intercepts.
B) Find the vertical asymptotes.
C) Find the slant asymptote.
D) Find the intervals of increase and the intervals of decrease.
E) Find the local maxima and minima.
F) Use the values you found in parts A through E to sketch the graph.

12. A particle is moving along the curve $y=x^2$. As the particle passes through the point (2,4), its x-coordinate increases at a rate of 2 $\frac{\text{cm.}}{\text{sec.}}$. How fast is the distance from the particle to the origin changing at this instant?

13. A right circular cylinder is inscribed in a sphere of radius 3 cm. Find the largest possible volume of such a cylinder.



Answers:

1. **A**
2. **B**
3. **F**
4. **E**
5. **E**
6. **F**
7. **B**
8. **C**
9. **A**
10. **E**
11.

a) $(0,0)$ and $(-1,0)$

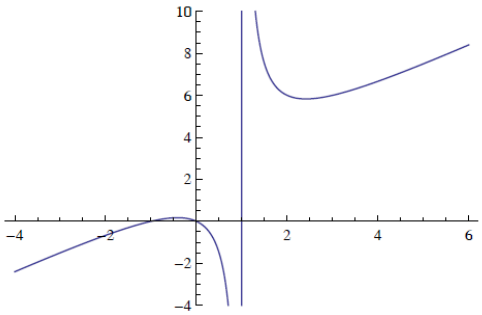
b) $x=1$

c) $y=x+2$

d) Increasing: $(-\infty,1-\sqrt{2})\cup(1+\sqrt{2},\infty)$ Decreasing: $(1-\sqrt{2},1)\cup(1,1+\sqrt{2})$

e) Local Max Value: $3-2\sqrt{2}$ Local Min Value: $3+2\sqrt{2}$

f)



12. $\frac{18\sqrt{5}}{5}\ \frac{\text{cm.}}{\text{sec.}}$

13. $12\pi\sqrt{3}\ \text{cm.}^3$