

## Math 103

### Final Exam

### Fall 2009

1. If  $\int_0^3 f(x) dx = 12$  and  $\int_0^6 f(x) dx = 42$ , find the value of  $\int_3^6 (2f(x) - 3) dx$ .

- a) 50          b) 51          c) 52          d) 56  
e) 53          f) 54          g) 55          h) None of these

2. Find the value of the integral  $\int_0^8 \sqrt[3]{x} \left( \frac{1}{\sqrt[3]{x^2}} + 1 \right) dx$ .

- a) 14          b) 6          c) 12          d) 20  
e) 18          f) 72          g)  $\frac{5}{2}$           h) 168

3. Find the value of  $\int_e^{e^2} \frac{(\ln x)^2}{x} dx$ .

- a)  $\ln 2$           b)  $\frac{1}{2} \ln 2$           c)  $\frac{1}{2}$           d)  $\frac{3}{2}$   
e) 1          f)  $\frac{1}{\ln 2}$           g) 0          h)  $\frac{7}{3}$

4. Find the value of  $\int_0^{\ln 9} e^{x/2} dx$ .

- a) 4          b) 2          c) 6          d) 3  
e) 16          f) 9          g) 8          h) 1

5. Find all critical numbers for the function  $f(x) = \sqrt[3]{9 - x^2}$ .

- a) 0          b) -3          c) 0, -3          d) No critical numbers  
e) 3          f) 3, -3          g) 0, 3, -3          h) None of these

6. At what value(s) of  $x$  is the function  $f(x) = \begin{cases} x^2 + 4x + 5 & \text{if } x < -2 \\ \frac{1}{2}x & \text{if } -2 \leq x \leq 2 \\ 1 + \sqrt{x-2} & \text{if } x > 2 \end{cases}$  discontinuous?

- a) -2            b) 0            c) -2,0, and 2            d) -2 and 0  
 e) 2            f) -2 and 2            g) 0 and 2            h)  $f$  is continuous everywhere

7. Find the interval on which the graph of  $f(x) = \ln(x^2 + 1)$  is concave upward.

- a) (-1,1)            b) (-1,2)            c) (-2,1)            d) (-2,2)  
 e) (-1,3)            f) (-3,2)            g) (-3,3)            h)  $(-\infty, \infty)$

8. The curve  $y = x^3 + x^2 - x$  has two horizontal tangents. Find the distance between these two horizontal lines.

- a)  $\frac{11}{9}$             b)  $\frac{22}{27}$             c)  $\frac{32}{27}$             d)  $\frac{5}{3}$   
 e)  $\frac{14}{9}$             f)  $\frac{4}{3}$             g)  $\frac{13}{9}$             h)  $\frac{7}{3}$

9. If  $f(x) = \frac{x}{\tan x}$ , find  $f'\left(\frac{\pi}{4}\right)$ .

- a)  $\frac{2-\pi}{2}$             b)  $\frac{1-\pi}{2}$             c)  $1-\pi$             d)  $\frac{\pi}{2}$   
 e)  $1-2\pi$             f)  $2-\pi$             g)  $2-2\pi$             h)  $-\pi$

10. Evaluate the limit  $\lim_{x \rightarrow \infty} \frac{\ln(3+2e^{5x})}{6x}$ .

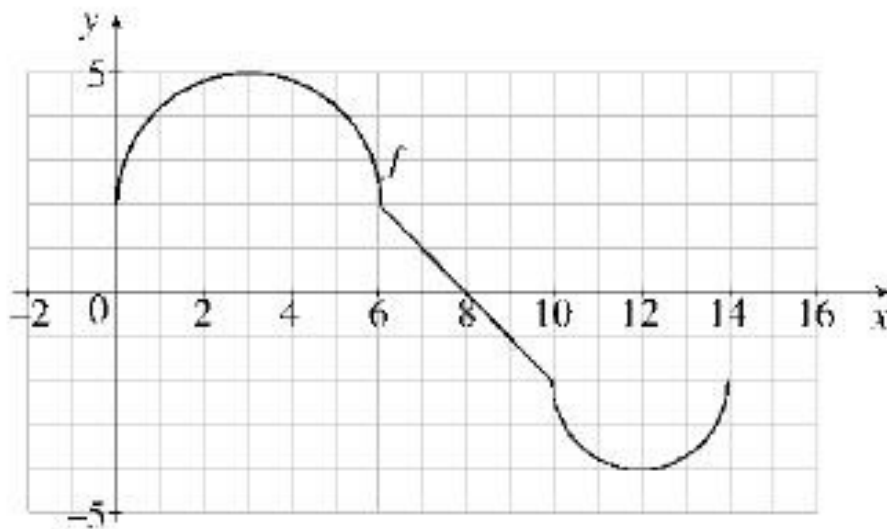
- a) 5            b)  $\frac{1}{5}$             c)  $\frac{5}{3}$             d)  $\frac{3}{5}$   
 e)  $\frac{1}{6}$             f)  $\frac{1}{10}$             g) 10            h)  $\frac{5}{6}$

11. Let  $V$  be the volume of a cylinder having height  $h$  and radius  $r$ , and assume that  $h$  and  $r$  vary with time. When the height is 5 in. and is increasing at 0.2 in./s., the radius is 3 in. and is decreasing at 0.1 in./s. How fast is the volume changing at that instant?

12. A rectangle with base on the  $x$ -axis has its upper vertices on the curve  $y = 12 - x^2$ . Find the maximum area of such a rectangle. Be sure to prove that you have found the maximum area.

13. The graph of  $f$  below consists of line segments and semicircles. Let  $g(x) = \int_0^x f(t) dt$ .

Answer the following questions.



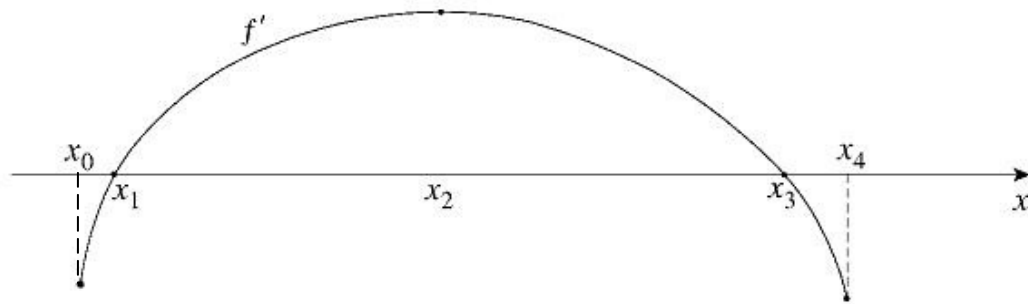
(a)  $g(14)$

(b)  $g(10)$

(c)  $g'(6)$

(d) What is the absolute minimum value of  $g$  on the interval  $[0, 14]$ ?

14. Given the graph of  $y = f'(x)$ , answer the questions that follow.



(a) Find all values of  $x$  at which (Explain your answers for full credit)

(i)  $f$  is increasing.

(ii)  $f$  is decreasing.

(iii)  $f''(x) > 0$ .

(iv)  $f$  has an inflection point.

(v)  $f$  has a local maximum

(b) Sketch a graph which could represent  $y = f(x)$ .

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### Answers

1. B

2. E

3. H

4. A

5. G

6. A

7. A

8. C

9. A

10. H

11.  $\frac{-6\pi}{5}$

12.  $32 \text{ in.}^2$

13. a)  $4 + \frac{5\pi}{2}$       b)  $12 + \frac{9\pi}{2}$       c) 2      d) 0

14. a)

i)  $(x_1, x_3)$     ii)  $(x_0, x_1) \cup (x_3, x_4)$     iii)  $(x_0, x_2)$     iv)  $x_2$     v)  $x_3$

b)

