

Math 103 Fall 2007
Final Exam
December 12, 2007

NAME _____
Student ID # _____

Circle the name of your professor

CROTTY

STORM

RIMMER

Work all problems in the space provided. You may use the back of each sheet for additional space. Please indicate when you do so.

There are 15 free-response partial credit questions. Please write legibly so that the proper credit may be given. You must show all work, an unsupported answer will receive little or no credit.

For grading purposes only. Do NOT write below this line.

	Points
1	
2	
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15	
Total	

1. $f(x) = 2x^3 + x^2 - 20x + 1$ has a local minimum at what x -value?

2. Find the equation of the tangent line to the graph of $f(x) = 2x^3 + 4x^2 - 5x - 3$ at the point $(-1, 4)$.

3. Let $f(x) = x^2 \cos^2(3x)$. Find $f'\left(\frac{\pi}{3}\right)$.

4. Determine where the function $f(x) = x + 2 \cos x$ is decreasing on $[0, 2\pi]$

5. Find the x -coordinate(s) of the inflection points of the function

$$f(x) = 6x^4 + x^3 - 3x^2 + 3x + 5$$

6. (a)

$$f(x) = \begin{cases} cx^2 - 3 & \text{if } x \leq 2 \\ cx + 2 & \text{if } x > 2 \end{cases}$$

f is continuous provided c equals what value?

(b) Sketch a function that has the following qualities:

$$f(2) = -3$$

$$f(5) = 4$$

f is continuous on $(0, \infty)$

$f(2)$ is a local minimum

$f(5)$ is neither a local minimum nor a local maximum

$f'(2)$ is undefined

$f'(5)$ is undefined

7. Evaluate

a) $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x^2 - 5x + 6}$

b) $\lim_{x \rightarrow -2} \frac{|x+2|}{x+2}$

c) $\lim_{x \rightarrow \infty} \frac{4x^3 + 2x - 4}{4x^2 - 5x + 6x^3}$

8. Evaluate.

a) $\int_0^1 \left(\frac{x^2 + 1}{\sqrt[3]{x}} \right) dx$

b) $g(x) = \int_0^{\frac{1}{2}x^4} \sin^3 t \cos t dt$. Find $g'(x)$

c) $\int_0^{\sqrt{\pi}} x \sin(x^2) dx$

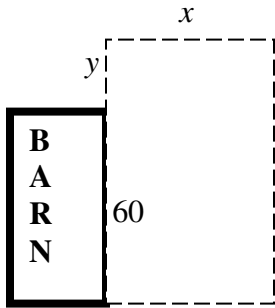
9. Let $r(t)$ be the rate at which the world's natural gas supply is being consumed, where t is measured in years starting at $t = 0$ on January 1, 2002, and $r(t)$ is measured in barrels per

year. What does $\int_0^3 r(t) dt$ represent?

10. Find the area of the region bounded by the parabolas $y = 2x - x^2$ and $y = x^2$

11. Find the volume of the solid of revolution obtained by revolving the region between the curve $y = 2x - x^2$ and the x -axis from $x = 0$ to $x = 2$ about the x -axis.

12. A farmer wants to construct a pen next to a barn 60 feet long, using all of the barn as part of one side of the pen. Find the dimensions of the pen with the largest area that the farmer can build if 300 feet of fencing material is available. There is no fence along the barn wall.



13. A weather balloon is rising vertically at the rate of 5 meters per second. An observer is standing on the ground 300 meters from the point where the balloon was released. At what rate is the distance between the observer and the balloon changing when the balloon is 400 meters high?

14. a)

x	-1	0	1	2	3
f	3	3	1	0	1
g	1	2	2.5	3	4
f'	-3	-2	-1.5	-1	1
g'	2	3	2	2.5	3

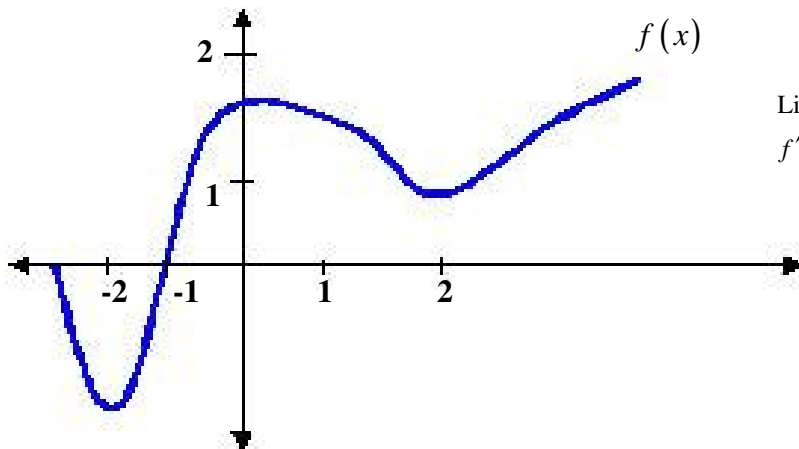
$$h(x) = f(x) \cdot g(x)$$

Find $h'(2)$

$$p(x) = f(g(x))$$

Find $p'(-1)$

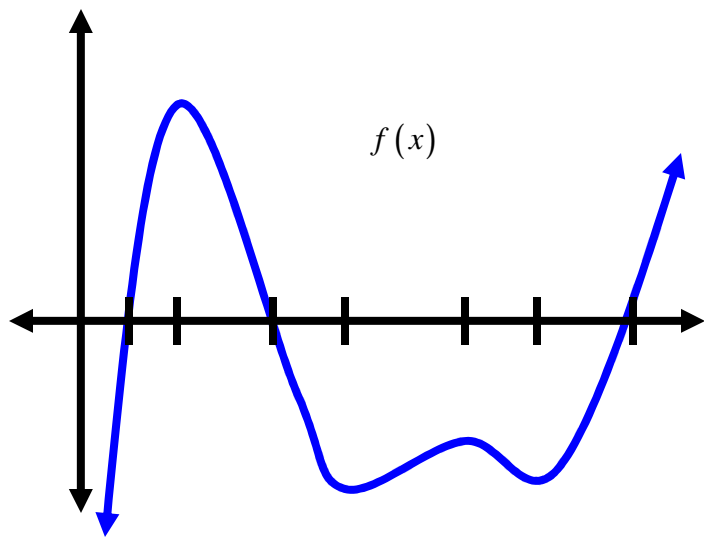
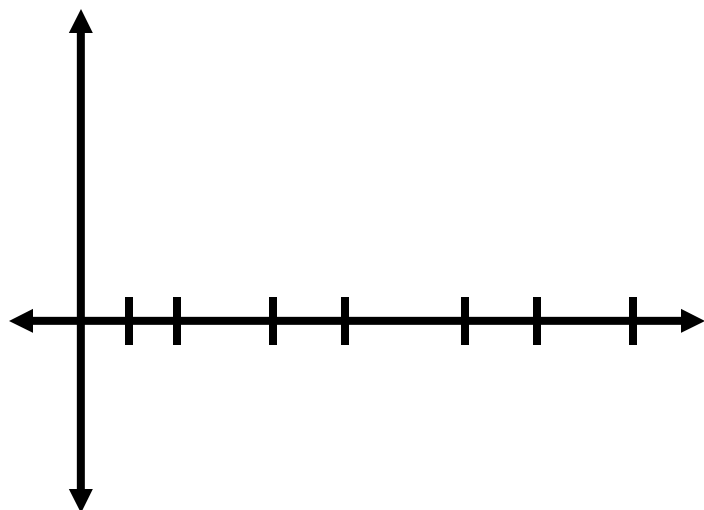
b)



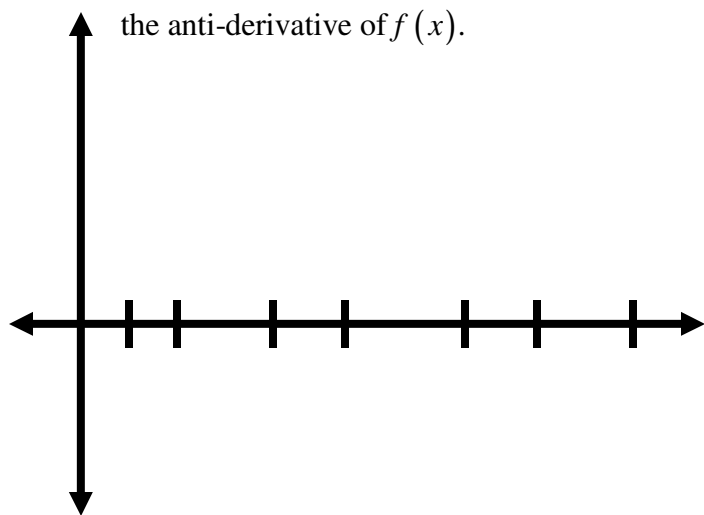
List in increasing order (from smallest to largest)
 $f'(-2)$ $f'(-1)$ $f(2)$ $f''(0)$

15.

Sketch the graph of $f'(x)$



Sketch the graph of $F(x)$
the anti-derivative of $f(x)$.



SOLUTIONS:

1. $\frac{5}{3}$

2. $y = -7x - 3$

3. $\frac{2\pi}{3}$

4. $\frac{\pi}{6} \leq x \leq \frac{5\pi}{6}$

5. $x = \frac{1}{4}$ and $x = \frac{-1}{3}$

6. a) $c = \frac{5}{2}$

b)

7.

a) -4

b) *dne*

c) $\frac{2}{3}$

8.

a) $\frac{15}{8}$

b) $\sin^3\left(\frac{1}{2}x^4\right)\cos\left(\frac{1}{2}x^4\right)2x^3$

c) 1

9. The total amount of natural gas consumed between January 1, 2002 and January 1, 2005 measured in barrels.

10. $\frac{1}{3}$

11. $\frac{16\pi}{15}$

12. 90 ft. \times 90 ft.

13. 4 $\frac{m}{s}$

14. a)

$$h'(2) = -3$$

$$p'(1) = -3$$

b) $f''(0) < f'(2) < f(2) < f'(-1)$

15.

