MATH 103 FINAL EXAM Fall 2003

1. $\int_1^e \frac{2\ln x}{x}\,dx =$

(a) 1 **(b)** 2 **(c)** e **(d)** $\frac{e^2}{2}$ **(e)** $e - \ln 2$ **(f)** $2e$

2. The point (4,2) is on the graph of $x = y^3 + 7y - 18$. What is $\frac{dy}{dx}$ there?

(a) 0 **(b) 1** **(c) 19** **(d) 1/19** **(e) 7** **(f) 1/7**

3. The graph of $y = x^2e^{4x}$ has (choose *all* that apply):

(a) a local minimum at $x = -2$
(b) a local maximum at $x = -2$
(c) a local minimum at $x = 0$
(d) a local maximum at $x = 0$
(e) a local minimum at $x = -1/2$
(f) a local maximum at $x = -1/2$

4. $\lim_{h\to 0} \frac{\ln(e^2 + h) - 2}{h} =$

(a) 2 (b) e^2 (c) $\frac{1}{e^2}$ (d) $\frac{1}{2}$ (e) 0 (f) ∞

5. One of the following lines through the origin is also tangent to the graph of $y = \frac{1}{10 - x}$. Which one? (*Hint*: Start by finding the equation of the tangent line to the graph at a generic point $(a, \frac{1}{10 - a})$.)

(a) $y = x$ (b) $y = x/5$ (c) $y = 10x$ (d) $y = x/10$ (e) $y = x/100$ (f) $y = x/25$

6. Evaluate (i) $\lim_{x\to 0} x \sin\frac{1}{x}$ and (ii) $\lim_{x\to\infty} x \sin\frac{1}{x}$

(a) (i) 1, (ii) 1 (b) (i) 0, (ii) ∞ (c) (i) 1, (ii) ∞
(d) (i) ∞, (ii) 0 (e) (i) 0, (ii) 1 (f) (i) ∞, (ii) 0

7. On which of the following intervals is the function $f(x) = \int_0^x e^{3t-t^3}\,dt$ concave up?

(a) $-1 < x < 1$ (b) $-\infty < x < -1 \cup 1 < x < +\infty$ (c) $-\infty < x < 0$

(d) $0 < x < +\infty$ (e) $-\sqrt{3} < x < \sqrt{3}$ (f) $\ln 2 < x < +\infty$

8. A 10-foot ladder is leaning against a vertical wall. If the bottom of the ladder slides away from the wall at a speed of 4 ft/sec, how fast, in radians per second, is the angle between the ladder and the wall (at the top of the ladder) increasing when that angle is $\pi/3$ radians?

(a) $\dfrac{10}{\sqrt{3}}$ (b) $\dfrac{\sqrt{3}}{10}$ (c) $\dfrac{4}{5}$ (d) $\dfrac{5}{2}$ (e) $\dfrac{\sqrt{3}}{5}$ (f) $\dfrac{10}{3}$

9. A cylindrical container is to be constructed having total volume 27 cubic feet. The sides are to be made of a special material that costs \$2 per square foot, while the material for the top and bottom costs only \$1 per square foot. What is the relationship between the radius r and the height h of the cheapest container that can be constructed according to these specifications?

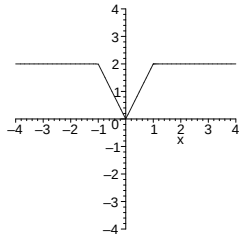
(a) $h = 3r$ (b)$r = 3h$ (c) $h = 2r$ (d) $r = 2h$ (e) $h = r$ (f) $h = 4r$

10. Let $f(x)$ be a function with the properties that $f'(x) > 0$ for all x, and that $f(0) = 1$, $f(1) = 2$, $f(3) = 5$ and $f(4) = 8$. Which of the following must be true? (*Hint*: Draw a picture!)

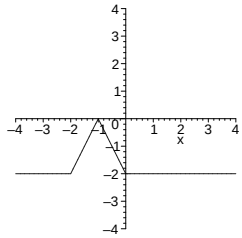
(a) $\int_0^4 f(x)\,dx > 20$ (b) $\int_0^4 f(x)\,dx < 20$ (c) $\int_0^4 f(x)\,dx < 16$

(d) $\int_0^4 f(x)\,dx > 16$ (e) $\int_0^4 f(x)\,dx < 10$ (f) $\int_0^4 f(x)\,dx > 24$

11. Here is the graph of a certain function, $y = f(x)$:



Here is a graph of a related function:



Which of the following is the equation of this graph?

(a) $y = f(x+1)$ (b) $y = f(x+1) - 1$ (c) $y = -f(x+1)$
(d) $y = -f(x-1)$ (e) $y = 1 - f(x-1)$ (f) $y = f(x-1) - 1$

12. Suppose $f(x)$ is a continuous function and that $\int_0^3 f(x)\,dx = 7$. Calculate $\int_0^9 \frac{f(\sqrt{x})}{\sqrt{x}}\,dx$.

(a) 7 (b) $\sqrt{7}$ (c) $\frac{7}{2}$ (d) 21 (e) $\frac{21}{2}$ (f) $\frac{\sqrt{7}}{2}$

13. Find the total area between the graph of $y = x^3 - 6x^2 + 8x$ and the x-axis.

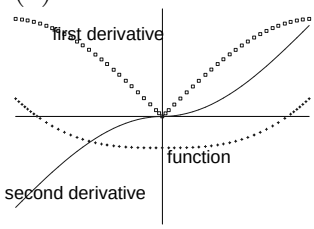
(a) 0 (b) 2 (c) 4 (d) 6 (e) 8 (f) 10

14. A solid is obtained by revolving the region between the lines $x = 1$ and $x = 5$, the x-axis, and the graph of $y = 1/(x+3)$ around the x-axis. What is its volume?

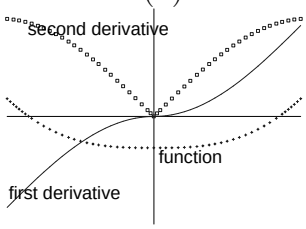
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{8}$ (c) 2π (d) $\frac{3\pi}{8}$ (e) $\frac{3\pi}{4}$ (f) $\frac{4\pi}{3}$

15. The following shows the graph of a function $f(x)$, together with the graph of its first derivative $f'(x)$ and its second derivative $f''(x)$. Each answer shows the same three graphs, but only one is labelled correctly. Which one?

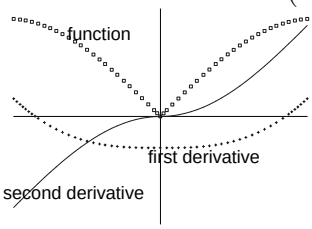
(a)



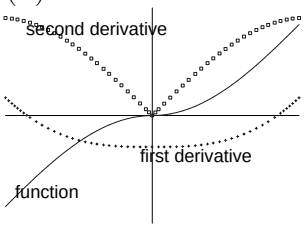
(b)



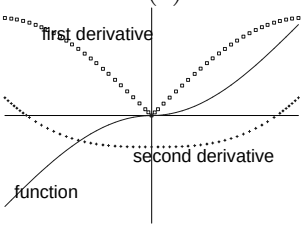
(c)



(d)



(e)



(f)