**Math 104 Final Exam — Spring 2011**

1. Consider the region that lies to the right of the $y$-axis and is enclosed between the graphs of $y = x^2$ and $y = 2x - x^3$. Which of the following integrals corresponds to the volume obtained when this region is rotated about the $x$-axis?

(a) $\pi \int_0^1 (2x - x^2 - x^3)\,dx$ (b) $\pi \int_0^1 (4x^2 - 5x^4 + x^6)\,dx$ (c) $\pi \int_0^1 x^2(2x - x^3)\,dx$

(d) $\pi \int_0^1 x(2x - x^2 - x^3)\,dx$ (e) $\pi \int_0^1 (4x^2 - 3x^4 + x^6)\,dx$ (f) $\pi \int_0^1 x^3(2 + x^4)\,dx$

2. A pyramid with a square base lies on the $x, y$-plane, with the vertices of its base at the points $(1, 1), (1, -1), (-1, 1), (-1, -1)$. The height of the pyramid is 2, and the vertex of the pyramid lies directly over the origin of the $x, y$-plane. What is the volume of the pyramid?

(a) 2 (b) 3 (c) 5/2 (d) 8/3 (e) 11/4 (f) 18/5

3. Find the average value of the function $\tan^2(x)$ over the closed interval $[0, \pi/4]$.

(a) $1 - \pi/4$ (b) $4/\pi - 1$ (d) $1/\pi - 1/4$ (d) $4/\pi$ (e) $\pi/4$ (f) 1

4. Evaluate $\displaystyle\int_e^{e^8} \frac{(\ln x)^{1/3}}{x}\,dx$.

(a) 12 (b) 15 (c) 16 (d) 20 (e) 45/4 (f) 64/3

5. Evaluate $\displaystyle\int_0^{\pi/2} x\sin(2x)\,dx$.

(a) $\pi/4$ (b) $\pi/3$ (c) $\pi/2$ (d) $\pi$ (e) $2\pi$ (f) $3\pi$

6. Evaluate $\displaystyle\int_0^1 \sin^3\left(\frac{\pi}{2}x\right)dx$.

(a) $2\pi/3$ (b) $4\pi/3$ (c) $2/3\pi$ (d) $1/3\pi$ (e) $3/4\pi$ (f) $4/3\pi$

7. Suppose that $f(x)$ is a function such that $f''(x) = \cos(x^3)$. The trapezoidal rule is then used to approximate the integral $\displaystyle\int_0^1 f(x)\,dx$, using ten subintervals of equal length. What is the strongest statement that can be made about the size of the error $E$, based just on the general error bound for approximations via the trapezoidal rule?

(a) $|E| \le 1/10^6$ (b) $|E| \le 1/8000$ (c) $|E| \le 1/1200$
(d) $|E| \le 1/300$ (e) $|E| \le 1/10$ (f) $|E| \le 1$.

8. The improper integral $\displaystyle\int_0^1 \frac{1}{(2x-1)^{\frac{1}{3}}}\,dx$

(a) $= 0$. (b) $= \frac{\ln(2)}{2}$. (c) $= -\frac{\ln(2)}{2}$. (d) $= 1 - \ln(5)$. (e) $= \frac{\ln(3)}{2}$. (f) diverges.

9. Find the arc length of the graph of $y = \dfrac{x^3}{3} + \dfrac{1}{4x}$ between $x = 1$ and $x = 2$. [Note: It may be helpful to use identities like $(x^2 + \frac{1}{4x^2})^2 = x^4 + \frac{1}{2} + \frac{1}{16x^4}$.]

(a) 0 (b) 59/24 (c) $\frac{8}{27}(10\sqrt{10} - 1)$ (d) $\pi \ln(2)$ (e) $\frac{3}{8} + \ln(2)$ (f) It is divergent.

10. Consider the graph of $y = \ln(\cos(x))$ between $x = 0$ and $x = 1$. Which of the following integrals corresponds to the surface area of the object obtained by rotating this graph about the $x$-axis?

(a) $\displaystyle\int_0^1 2\pi\sqrt{1 + \ln(\cos(x))^2}\,dx$ (b) $\displaystyle\int_0^1 2\pi \ln(\sin(x))\sqrt{1 + \sec^2(x)}\,dx$

(c) $\displaystyle\int_0^1 2\pi \cos(x) \ln(\sin(x))\,dx$ (d) $\displaystyle\int_0^1 2\pi \sec(x) \ln(\cos(x))\,dx$

(e) $\displaystyle\int_0^1 2\pi x^2 \sin(x) \cos(x) \ln(x)\,dx$ (f) $\displaystyle\int_0^1 2\pi \sin^2(x)\sqrt{1 + \ln(x)^2}\,dx$

11. A certain random variable has probability density function $f(x)$ given by $f(x) = xe^{-x}$ for $x > 0$, and $f(x) = 0$ for $x \leq 0$. Find the mean of this random variable.

(a) $-2$ (b) $-1$ (c) 0 (d) 1 (e) 2 (f) 3

12. Suppose that a sequence $\{a_n\}$ converges to $\pi$. Then the sequence $\{\cos(a_n)\}$

(a) converges to $-2$. (b) converges to $-1$. (c) converges to 0.
(d) converges to 1. (e) converges to $\pi$. (f) diverges.

13. Suppose that a series $\sum_{n=1}^{\infty} a_n$ converges to $e$, where each $a_n > 0$. Then the series $\sum_{n=1}^{\infty} 1/a_n$

(a) converges to $-1/e$. (b) converges to $1/e$. (c) converges to 0.
(d) converges to $e$. (e) converges to $\pi$. (f) diverges.

14. The series

$$\sum_{n=1}^{\infty} \frac{10n^3 2^{n+4}}{\pi^n}$$

(a) converges by the alternating series test.
(b) diverges by the alternating series test.
(c) converges by the ratio test.
(d) diverges by the ratio test.
(e) converges because the terms approach 0.
(f) diverges because the terms do not approach 0.

15. The series

$$\frac{1}{1^{3/2}}+\frac{1}{2^{3/2}}-\frac{1}{3^{3/2}}+\frac{1}{4^{3/2}}+\frac{1}{5^{3/2}}-\frac{1}{6^{3/2}}+\frac{1}{7^{3/2}}+\frac{1}{8^{3/2}}-\frac{1}{9^{3/2}}+\cdots$$

(a) converges because the terms approach 0.
(b) diverges because the terms do not approach 0.
(c) converges by the alternating series test.
(d) diverges by the alternating series test.
(e) converges by the absolute convergence test and $p$-test.
(f) diverges by the absolute convergence test and $p$-test.

16. The series $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{n}2^{n-1}x^n$ is convergent precisely for the following values of $x$:

(a) $-\frac{1}{2}<x\leq\frac{1}{2}$ (b) $0\leq x<4$ (c) $x\in\{-2,2\}$ (d) $-2<x\leq 2$ (e) $x=\pi$ (f) $x<5$

17. $\displaystyle\sum_{n=0}^{\infty}\frac{(\ln 2)^n}{n!}$

(a) $=\pi$. (b) $=e$. (c) $=2$. (d) $=4\ln(2)$. (e) $=\cos(2)$. (f) diverges.

18. Find the coefficient of $x^{10}$ in the Maclaurin series expansion of $f(x)=4-x\sin(x^3)$.

(a) 0 (b) 4 (c) 10 (d) 1/6 (e) 1/10 (f) 1/10!

19. Suppose that $y=f(x)$ satisfies the differential equation $\dfrac{dy}{dx}+\dfrac{y}{x+1}=y$, and also satisfies the intitial condition that $y=5$ when $x=0$. What is the value of $y$ when $x=3$?

(a) $e^3$ (b) $e^{-1}$ (c) $4e^{-3}/5$ (d) $4e^3/5$ (e) $5e^{-3}/4$ (f) $5e^3/4$

20. A certain population grows according to the differential equation

$$\frac{dP}{dt}=\frac{P}{20}(1-\frac{P}{4000})$$

and the initial condition $P(0)=1000$. What is the size of the population at time $t=10$?

(a) 1751 (b) $1000+20/e$ (c) $4000e^{1/20}$
(d) $1000+200e^{1/20}$ (e) $4000/(1+3e^{-1/2})$ (f) $1000/(1+20e^{-10})$