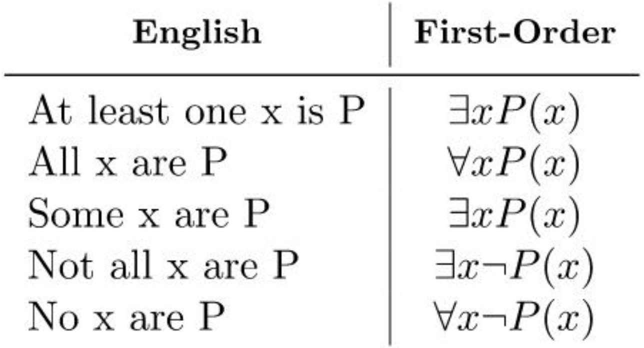
First Order Logic and Relational Databases

Quincy Alston

Logical Statements on Sets

A theory **T** is a set of first order logical sentences



English	First-Order
At least one x is P	$\exists x P(x)$
All x are P	$\forall x P(x)$
Some x are P	$\exists x P(x)$
Not all x are P	$\exists x \neg P(x)$
No x are P	$\forall x \neg P(x)$

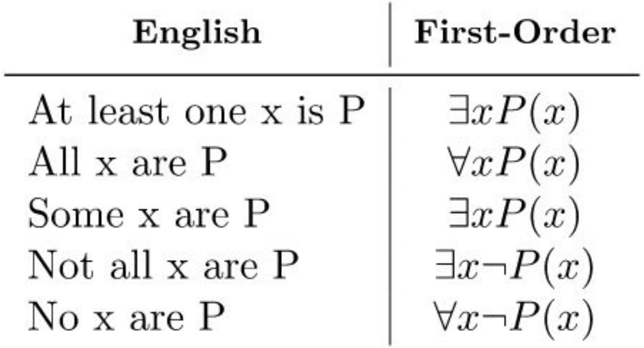
Image: Baeldung

Satisfaction relation: ⊨

A theory is satisfiable if there exists a model which satisfies all sentences in the theory i.e. **M**⊨**T**

Logical Statements on Sets

A theory **T** is a set of first order logical sentences



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Satisfaction relation: $\vDash$

A theory is satisfiable if there exists a model which satisfies all sentences in the set **T** i.e. $\mathbf{M} \vDash \mathbf{T}$

Define a set in a model **M**

$$\Phi[\mathbf{M}] \coloneqq \{a \in \mathbf{M} \mid \mathbf{M} \vDash \Phi(a)\}$$

Relational Databases

- Logic
  - “Data set”; Set is the universe
  - Predefined relationships on said set
- Relational
  - Set of data points that we are interested in
  - Only the points for which the relation is true
  - Universe: The disjoint union of the data types

City	**id**	**name**	**population**	**state**
	1	Philadelphia	500	PA
	2	San Francisco	300	CA
	3	New York	250	NY

Define a function **City** mapping to true or false

City(“Philadelphia”, 500, PA) is true

City(“New York”, 300, PA) is false

City	**id**	**name**	**population**	**state**
	1	Philadelphia	500	PA
	2	San Francisco	300	CA
	3	New York	250	NY

SQL

- Programming language - “Structured Query Language”
- Manipulation of/queries on relational databases
- SELECT: return columns on condition, * denotes every column
- FROM: points the table which we would like to query
- WHERE: condition on which to return columns
- IN: equivalent of logical or statements

Logical Projection

- Sets **X,Y**
- Product of sets **V=X×Y** of the form **(x,y)**
- Projection Function **Px** takes **X×Y** → **X** s.t. **Px(x,y)=x**
- Logical Function **Φ(x)≔ ∃ y**,**(x,y)⋵V**

$$\begin{tikzcd}
 & X\times Y\times Z \arrow[dl, "Px"] \arrow[d, "Py"'] \arrow[dr, "Pyz"] & \\
X & Y & Y\times Z
\end{tikzcd}$$

- SELECT: return columns on condition, * denotes every column
- FROM: points the table which we would like to query
- WHERE: condition on which to return columns
- IN: equivalent of logical or statements

SELECT * FROM City

Return all columns from the table “City” on the condition TRUE

$$\Phi(x,y,z) \coloneqq \mathbf{City(x,y,z)}$$

SELECT * FROM City

Return all columns from the table “City”

$\Phi(x,y,z) \coloneqq \mathbf{City(x,y,z)}$

SELECT name FROM City

Return name from the table “City”

$\Phi(x) \coloneqq \exists\, y, \exists\, z, \mathbf{City(x,y,z)}$

SELECT name FROM City WHERE state=”CA”

Return name from the table “City” on the condition state is “CA”

$$\Phi(x) \coloneqq \exists\, y, \exists\, z, \textbf{City(x,y,z)} \land \textbf{(z=”CA”)}$$

SELECT name FROM City WHERE state=”CA”

Return name from the table “City” on the condition state is “CA”

$$\mathit{\Phi}(x) \coloneqq \exists\, y, \exists\, z, \mathbf{City(x,y,z)} \land \mathbf{(z=\text{”CA”})}$$

SELECT name FROM City WHERE state IN (”CA”, “PA”)

Return name from the table “City” on the condition state is “CA” or “PA”

$$\mathit{\Phi}(x) \coloneqq \exists\, y, \exists\, z, \mathbf{City(x,y,z)} \land \mathbf{((z=\text{”CA”})} \lor \mathbf{(z=\text{”PA”}))}$$

Join

- We use JOIN to combine rows from two or more tables
- Often JOIN is used with ON to specify a condition on which rows should be joined

City	**id**	**name**	**population**	**state**
	1	Philadelphia	500	PA
	2	San Francisco	300	CA
	3	New York	250	NY

Person	**id**	**name**	**height**	**residentOf**
	1	Aira	5’8’’	2
	2	Jane	5’2’’	2
	3	Max	6’0’’	1

```
SELECT Person.name FROM City JOIN Person ON Person.residentOf = CIty.id
WHERE City.name = “Philadelphia”
```

Join the rows from City and Person where City id and Person residentOf are the same.
Then Return Person name from the table Person only where City name is Philadelphia.

$$\Phi(x) \coloneqq \exists\, \mathbf{u}, \exists\, \mathbf{y}, \exists\, \mathbf{z}, \mathbf{Person(x,u)} \wedge \mathbf{City(}\text{“Philadelphia”}\mathbf{,y,z)} \wedge \mathbf{(z=\text{”CA”})}$$

```
SELECT name FROM City WHERE state=”CA” AND population > 200
```

Return name from the table “City” on the condition state is “CA” and the population is above 200

$$\Phi(x) \coloneqq \exists\, y, \exists\, z, \mathbf{City(x,y,z)} \wedge \mathbf{(z=\text{”CA”})} \wedge \mathbf{(200<y)}$$