

Calculus Concept Inventory

The U-M Mathematics Department has for many years had a unique focus on Calculus teaching. The instructional methods and teacher training have been recognized over the years as innovative and effective among its peer institutions. This year the U-M participated in a test of conceptual gain in understanding using the Calculus Concept Inventory (CCI).

Background

The CCI was developed under an NSF grant by a panel of experts led by mathematician Jerome Epstein of Polytechnic University. The instrument is designed to test the concepts of differential Calculus with 22 multiple choice questions in the form of a pre-test and post-test. The normalized gain of the class is determined by the percent of correct answers using the following formula:

$$\text{Gain} = (\text{mean post-test score of class} - \text{mean pre-test score of class}) \div (100 - \text{mean pre-test score of class})$$

Studies on similar inventories used in other academic subjects, particularly Physics, show that the normalized gain is independent of students' prior knowledge, but highly related to teaching methodology. The Physics research has indicated that classrooms that are "interactively engaged" (*i.e.*, students are continually engaged in developing concepts and strategies to solve problems, testing solutions, and receiving immediate feedback), show more significant normalized gains than traditional lecture classrooms. Prior to the fall of 2008, the CCI had been administered to approximately 2000 students in 25 universities and colleges. The normalized gains from all schools were very low, ranging from 0.08 to 0.20. Exceptions to these results were from three specific instructors striving to teach in an interactively engaged (IE) style.

U-M Testing and Results

In the fall 2008 semester the CCI was administered to all sections of Calculus I at U-M. There were 51 sections, each with 32 or fewer students. Instructors in the course have full responsibility for teaching their individual sections. All new math instructors at U-M (faculty and graduate students) attend a week-long training workshop designed to help them teach in the IE style. For the semester, 96% of the students in Calculus I took both the pre- and post- CCI tests. On the post-test, students were asked to rate the perceived interactivity level of the classroom, and the percentage of time spent on interactively engaged activities.

The U-M Calculus program achieved outstanding results on the CCI. The average gain over all 51 sections was 0.35, and 10 sections had a gain of 0.40 to 0.44. These 10 highest scoring sections also had the highest scores for perceived interactivity, and were reported as spending the most time on interactively engaged activities.

U-M Calculus Training

It is clear that the IE style of teaching can be fostered and developed; over one third of the Calculus I sections at U-M in the Fall 2008 semester were taught by instructors new to the course. All new instructors within the Mathematics Department, including graduate students, post-doctoral faculty and tenure-track faculty, participate in a week-long training program that introduces the concepts of the IE classroom. This hands-on training includes activities for the classroom, actual role playing and practice, and handling challenges that may arise.

Karen Rhea, a senior lecturer with the Department, is the director of the Freshman-Sophomore program and manages the training program as well as the Calculus I course. "The instructor training is one of the significant factors in the success of our Calculus program," Rhea says. "I believe that this type of training is unique on campus." In addition, there are weekly meetings for the Calculus I instructors to discuss activities and provide feedback and support. While each instructor independently manages their sections of the course, there is a great deal of collaboration. From the initial training, the instructors bond both professionally and socially, helping them to acclimate to the Department.

The conceptual understanding of the students in all of the introductory courses is continually assessed through regular quizzes as well as standardized exams. "I am pleased that our students have shown significant conceptual gains as measured by the CCI" says Rhea. U-M Mathematics will continue to encourage the IE teaching style in Calculus I and other courses, in particular the Inquiry Based Learning courses. In all of these courses, there is an emphasis on the underlying *ideas* of the course as well as development of skills. In the related articles, two instructors share their experiences with learning and teaching Michigan Calculus.



Calculus students working on group projects

**Join us for the Michigan Reception at the
Joint AMS Meetings in San Francisco, CA,
Friday, January 15, 2010**

5:30–7:00 PM

Pacific Room J. San Francisco Marriott

www.math.lsa.umich.edu/alumni

Interactively Engaged (IE) Classrooms from the Instructor's View

Here a postdoc and graduate student share their experiences teaching Michigan Calculus in an IE classroom for the first time.

Christopher Mooney

Christopher Mooney joined the Mathematics Department in September 2008 as a postdoc assistant professor. He received his Ph.D. from UW-Milwaukee. His research is in the area of geometric group theory.



Coming into the calculus program at Michigan was a novel experience for me. Having taught for the previous six years using the traditional lecture style with its traditional focus on computations and proofs, the movement to this new paradigm was certainly challenging. I quickly became a convert, however, when I discovered the amazing fact that students really can learn underlying ideas. Students who are taught under the traditional lecture style may learn to compute the derivative or integral of a function, but more often than not they do not understand the reasoning. For the engineers and accountants going into the field, it is much more important that they understand the meaning of the derivative rather than simply how to compute it. But if they forget how to take the derivative and remember why they need to, they are much better off than if they simply remember the computation, which is something any computer can do.

For me there are two key new ideas in this nontraditional way of teaching calculus. The first is an increased emphasis on understanding and interpretation rather than simply on computation. The book we use is especially helpful in facilitating this. It has more narrative than books I have used in the past, and the thought-provoking problems usually had some real-world context. One of the problems which will remain etched in my memory as the “Classic Michigan Calculus Problem” is that of interpreting the derivative in the simplest possible terms. Of course the students find this exercise particularly challenging, but it was a very useful application of using derivatives to solve word problems.

The second new idea is the practical side of the first. Through a movement away from lecture and towards group work and open discussion in the classroom, students develop a deeper understanding of the subject. Students would work in groups, with me moving from group to group watching them struggle and discuss. I would find a student who had the correct answer and have them do it on the board. The students' natural nervousness abated when they discovered that they didn't have to get their answer perfect. As a class we would help them fix their mistakes and everyone would learn. The conceptual understanding of the subject is strengthened when

students use and experiment with new ideas immediately upon being introduced to them.

None of this is possible without the effective training that we received. I was definitely on board with the conceptual emphasis, but the movement towards group work and away from lecture I found quite uncomfortable. The trainers emphasized the importance of it and modeled it for us, which helped immensely in implementation. I discovered that the students were much more engaged with the concepts than when they simply watched me do it on the board. I now find this teaching style to be more effective and enjoyable for both parties.

Geoffrey Scott

Geoffrey Scott joined the Mathematics Department as a graduate student in 2008. He received his undergraduate degree from Dartmouth College and is interested in studying topology.

Throughout the week of teacher training, the program directors make it clear that calculus is taught differently at Michigan. Through role-playing sessions, we learn not only the basics of lecturing, but also techniques to make class engaging and interactive, and ways to emphasize conceptual understanding above mere symbol manipulation.

Like all instructors in the calculus sequence, I lecture for only about half of the allotted class time. During the other half, the class works on problems while I visit with small groups of students at a time. This practice helps students internalize the lesson before class ends. At most schools, a student could attend a lecture on Monday and be unaware they need help until they attempt their homework several days later. At Michigan, since students work on problems during class, we can immediately resolve any confusions that may arise.

Towards the goal of emphasizing conceptual understanding, Michigan has adopted non-traditional syllabi and assignments for the calculus sequence. To be sure that students are absorbing the concepts, we require them to work weekly with a group of peers on difficult problems, and to hand in not just their solutions but also a detailed account of the reasoning behind their work. Since most groups arrive at the correct numerical solution to each problem, their homework grade is based mostly on the soundness and completeness of their reasoning.

Many incoming students expect their intro calculus class to be lifeless. By making class a cooperative and interactive experience, we ensure that they're engaged. I feel that most students leave the class understanding the material on a quite deep level. It makes the class very satisfying to teach.