107: Probability, Functions, Sequences

1. Combinatorics
   - Cartesian Products and Power Sets (review)
   - Combinations and Permutations
     - factorials and permutations of an n-element set
     - squads and notation: $\binom{n}{k}$
     - tickets and notation P_k^n
   - Binomial Coefficients
     - formula in terms of factorials
     - Pascal's triangle
     - justification of rule for generating Pascal's triangle
2. Probability
   - Probability Spaces
     - equally likely outcomes
     - meaning of probability: probability models
     - probabilities as fractions
     - probabilities of unions and complements
   - Independence and Conditional Probability
     - intuitive concept
     - product law
     - product spaces: why is $\mathbf{P}$(one head out of two flips) $= \frac{1}{2}$ and not $\frac{1}{3}$?
     - conditional probability
     - word problems calling for conditional probability
   - Combinatorial Probability
     - k heads out of n coins
     - sums of dice
     - applications: poker hands, lottery
   - Law of Large Numbers
     - gambler's fallacy
     - strong law and correction of fallacy
     - weak law verified by Pascal's triangle
3. Functions
   - General Functions
     - function machines
     - $f(x)$ notation
     - writing formulae for functions
     - addition, multiplication of functions
     - composition of functions
   - Famous Functions
     - polynomials and their properties
     - the exponential function (see Compound Interest: instantaneous compounding, below)
     - the logarithm and logarithmic scales
4. Sequences and Series
   - General Sequences
     - subscript notation
     - sequences defined by formula
     - sequences defined recursively

  - use of spreadsheet to compute these
  - arithmetic sequences: how to recognize, how to write a formula
  - geometric sequences: how to recognize, how to write a formula
  - Fibonacci and other famous sequences
- Series
  - summation notation
  - free and bound variables
  - summing arithmetic and geometric series
  - summing via spreadsheet

5. Logic
   - Review of Propositional Logic: the converse and contrapositive
   - Quantifiers
     - universal and existential quantifiers
     - translating between English and formally quantified statements
     - what does it take to verify a quantified statement?
     - syllogisms
   - Analysis of Arguments
     - proving a compound proposition, including an if-then statement
     - proving a quantified statement
     - proof by contradiction
     - proving the contrapositive
     - how to recognize an invalid argument
   - Classic Proofs
     - irrationality of $\sqrt{2}$
     - infinitude of primes
6. Compound Interest
   - What is Compound Interest?
     - simple versus compound
     - APR
     - instantaneous compounding and the exponential
     - writing it as a sequence or a function
   - Computing
     - formula relating initial amount, interest rate, time and balance
     - applications: mortgages, car loans, annuities
     - use of spreadsheets for computation
7. General Skills
   - Estimation and Mental Calculation
     - approximating to one or two significant figures
     - algebraic tricks for mental computation
     - familiarity with orders of magnitude of some common large quantities
   - Recursive Use of Notation
     - composition of functions
     - functions of complicated arguments
     - expressions appearing as indices of summation
   - Broader Context
     - relevance of argument analysis to classroom learning
     - NCTM Standards for rigor and quantitative reasoning