

Problem 1, solution 1: Let's rename the points C', E' and F' , which are the perspective images of some collinear real world points C, E and F . If $|CE| = 2|EF|$ then the points C, E, F, ∞ have cross ratio 2. In fact this is if and only if. Since cross ratio is a projective invariant, we can measure and test whether C', E', F', V has cross ratio equal to 2.

Problem 1, solution 2: Denote the bottom endpoints of the crossbars with top endpoints C', E', F' by X', Y', Z' respectively. Intersect the diagonals $C'Y'$ and $E'X'$ to find the center point A' of quadrilateral $C'E'Y'X'$. Also find the vanishing point W for the crossing segments $C'X', E'Y'$ and $F'Z'$ (these are images of parallel segments CX, EY and FZ so they have a common vanishing point). The ray WA' is the image of a real life line dividing the real life rectangle $CEYX$ in half, meaning that we need to determine whether or not real life $EFZY$ is congruent to these halves. The easiest way to do this is to check whether the diagonals of the quadrilateral $A'F'Z'Y'$ intersect at a point on the segment $E'Z'$.

In the picture below, we see that the diagonals intersect a bit to the left of $E'Z'$, so no, the first tile is not double the second.

